Guaranteed Bounds for Solution of Parameter Dependent System of Equations

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Abstract

Many scientific and engineering problems lead to the solution of parameter dependent system of equations in the form $A(p)u = b(p)$ where $p = (p_1, \ldots, p_m)$ is a vector of parameters. In the case when parameters are known only in some intervals $[\underline{p}_1, \overline{p}_1], \ldots, [\underline{p}_m, \overline{p}_m]$ then the solution of the parameter dependent system of equation is very complicated $u(P) = \{u : A(p)u = b(p), p \in P\}$. Approximate value of the extreme values of the solutions $u_i = \min\{u_i : u \in u(P)\}, \quad \pi_i = \max\{u_i : u \in u(P)\}$ can be calculated by using known optimization methods. In many cases the approximate solutions of the optimization problems are exact. In such situations, if the solution is a union of path-connected sets and the function $u = u(p)$ is continuous, in some cases, it is possible to prove that the approximate solution is exact. The method can be also applied for finding extreme values of the functions that depends on the interval solution, i.e. preprocessing. Postprocessing is very important in the practical applications. The method can be also applied for the solution of the eigenvalue problem with uncertainty.