Topological derivative and its applications to the modeling of uncertainty and optimization

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Many problems of computational science depend on the shape of certain sets Ω. In order to describe them it is necessary to apply set dependent functions e.g. $\psi = \psi(\Omega)$. Very often precise description of the set Ω is not known exactly. In such situations it is possible to estimate the shape of the set by using interval set $\Omega \in [\Omega^l, \Omega^u]$. Values of the function ψ are not known exactly. Upper and lower bounds of the function ψ can be defined as

$$\psi^l = min\{f(\Omega) : \Omega \in [\Omega^l, \Omega^u]\}, \quad \psi^u = max\{f(\Omega) : \Omega \in [\Omega^l, \Omega^u]\}. \tag{1}$$

Extreme values ψ^l, ψ^u can be calculated by using optimization method. One of the simplest and most efficient optimization method, which can be applied in this case is the gradient method which utilize the concept of topological derivative. In the presentation a review of different concepts of the topological derivative will be given.

In the case of engineering applications the function ψ is a solution of the system of partial differential equations, which after discretization can be written as

$$K\psi = Q \tag{2}$$

Sensitivity of of the solution $\frac{d\psi}{d\Omega(x)}$ can be calculated by using direct differentiation method from the following system of equations

$$K\frac{d\psi}{d\Omega(x)} = \frac{dQ}{d\Omega(x)} - \frac{dK}{d\Omega(x)}\psi. \tag{3}$$

Above described procedure will be applied to the modeling of uncertainty in structural mechanics and heat transfer problems.