

Algebraic Expressions with Uncertain Syntax and Their Applications in Online Learning

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Outline

- 1 Motivation
- 2 Online learning
- 3 How to create online assignments?
- 4 Typical problems in mathematical representation of the student's answers
- 5 Arithmetic expressions with errors
- 6 Multi-valued logic
- 7 Learning
- 8 Conclusions

Motivation

Motivation

Online
learning

How to create
online
assignments?

Typical
problems in
mathematical
representation
of the
student's
answers

Arithmetic
expressions
with errors

Multi-valued
logic

Learning

Conclusions

What to do with the student's answers with errors?

Correct answer:

$$(1 + 2)$$

Answers with errors:

$$(1 + 2($$

$$(1 + *2$$

$$(1 + (2)$$

...

Online learning

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Online learning

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Blackboard


 Institution Page

 Andrew Powinuk


 Activity Stream


 Courses


 Organizations


 Calendar

 Messages

 Grades

 Tools

 Sign Out



The University of Texas at El Paso

Blackboard Help




Enter Blackboard Central's virtual office Mon - Fri 8AM - 5PM
24/7 Blackboard Support 915-747-HELP option # 2
Blackboard Help Website
Provide your Blackboard feedback

Technology Support Helpdesk

MON-THU 7AM-9PM FRI 7AM-8PM SAT 9AM-2PM SUN 12PM-5PM
Visit us at helpdesk.utep.edu
Submit a request
Chat with us
Contact us: 915-747-HELP, helpdesk@utep.edu

TECHNOLOGY Support



Early examples of distance learning

Motivation

Online learning

How to create online assignments?

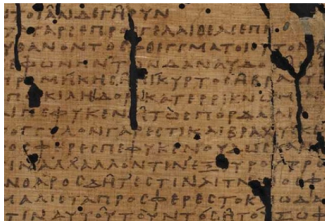
Typical problems in mathematical representation of the student's answers

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Online learning

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User Main Page

	Logoff
Username:	80080081
First Name:	Pownuk
Last Name:	Andrew
Group:	2020-Fall-MATH-1312-CRN-12219
	Change password
	Change e-mail

Show my grades

Files (notes, syllabus etc.)

List of students

Motivation

Online learning

How to create online assignments?

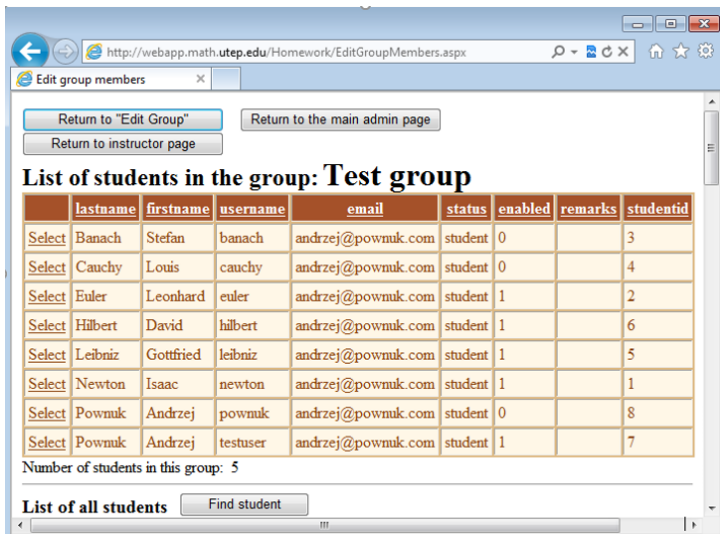
Typical problems in mathematical representation of the student's answers

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The screenshot shows a web browser window with the URL `http://webapp.math.utep.edu/Homework/EditGroupMembers.aspx`. The page title is "Edit group members". At the top, there are three buttons: "Return to 'Edit Group'", "Return to the main admin page", and "Return to instructor page". Below these buttons, the heading "List of students in the group: Test group" is displayed. A table lists the students in the group. Each row has a "Select" link in the first column, followed by columns for "lastname", "firstname", "username", "email", "status", "enabled", "remarks", and "studentid". Below the table, it says "Number of students in this group: 5". At the bottom, there is a section titled "List of all students" with a "Find student" button.

	lastname	firstname	username	email	status	enabled	remarks	studentid
Select	Banach	Stefan	banach	andrzej@pownuk.com	student	0		3
Select	Cauchy	Louis	cauchy	andrzej@pownuk.com	student	0		4
Select	Euler	Leonhard	euler	andrzej@pownuk.com	student	1		2
Select	Hilbert	David	hilbert	andrzej@pownuk.com	student	1		6
Select	Leibniz	Gottfried	leibniz	andrzej@pownuk.com	student	1		5
Select	Newton	Isaac	newton	andrzej@pownuk.com	student	1		1
Select	Pownuk	Andrzej	pownuk	andrzej@pownuk.com	student	0		8
Select	Pownuk	Andrzej	testuser	andrzej@pownuk.com	student	1		7

Number of students in this group: 5

List of all students

Information about particular student

Motivation

Online learning

How to create online assignments?

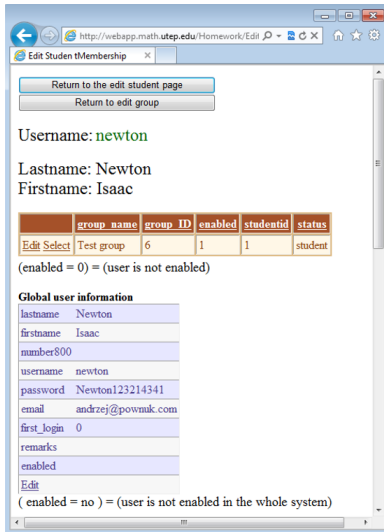
Typical problems in mathematical representation of the student's answers

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The screenshot shows a web browser window with the address bar displaying `http://webapp.math.utep.edu/Homework/Edit`. The page title is "Edit Student Membership". At the top, there are two buttons: "Return to the edit student page" and "Return to edit group". Below these, the student's information is displayed: Username: newton, Lastname: Newton, and Firstname: Isaac. A table follows, showing a list of groups. The table has columns: group_name, group_ID, enabled, studentid, and status. The first row shows "Test group" with group_ID 6, enabled 1, studentid 1, and status "student". Below the table, a note states "(enabled = 0) = (user is not enabled)". Under the heading "Global user information", a table lists user details: lastname (Newton), firstname (Isaac), number800, username (newton), password (Newton123214341), email (andrzej@pownuk.com), first_login (0), remarks, and enabled. An "Edit" link is provided at the bottom of this section. A final note at the bottom of the page states "(enabled = no) = (user is not enabled in the whole system)".

Return to the edit student page

Return to edit group

Username: newton

Lastname: Newton

Firstname: Isaac

	group_name	group_ID	enabled	studentid	status
Edit Select	Test group	6	1	1	student

(enabled = 0) = (user is not enabled)

Global user information

lastname	Newton
firstname	Isaac
number800	
username	newton
password	Newton123214341
email	andrzej@pownuk.com
first_login	0
remarks	
enabled	

[Edit](#)

(enabled = no) = (user is not enabled in the whole system)

Online homework

Motivation

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The screenshot shows a web browser window with the URL `http://webapp.math.utep.edu/Homework/EditHomeworkList.aspx`. The page displays a table of homework assignments. The table has four columns: an action column with 'Edit' and 'Select' links, an ID column, a title column, and a file path column. The assignments are numbered 42 through 50. Assignment 50, 'AA-I-Homework-15', is highlighted in blue. Below the table is a pagination bar with '1 2 3 4'. Below the table is a problem box with the text 'Find the Laurent series for' followed by the function $f(z) = \frac{\sin(2z)}{z^2}$ and 'at $z_0 = 0$ '. There are two input fields for coefficients: $a_{-1} =$ and $a_0 =$. At the bottom of the problem box is a button labeled 'Calculate and submit grade'.

	ID	Title	File Path
Edit Select	42	TG-Homework-2	/HomeworkDir/TG-Homework-2.aspx
Edit Select	43	Cal-III-Homework-13	/HomeworkDir/Cal-III-Homework-13.aspx
Edit Select	44	Cal-II-Homework-12	/HomeworkDir/Cal-II-Homework-12.aspx
Edit Select	45	AA-I-Homework-12	/HomeworkDir/AA-I-Homework-12.aspx
Edit Select	46	Cal-III-Homework-14	/HomeworkDir/Cal-III-Homework-14.aspx
Edit Select	47	Cal-II-Homework-13	/HomeworkDir/Cal-II-Homework-13.aspx
Edit Select	48	AA-I-Homework-13	/HomeworkDir/AA-I-Homework-13.aspx
Edit Select	49	AA-I-Homework-14	/HomeworkDir/AA-I-Homework-14.aspx
Edit Select	50	AA-I-Homework-15	/HomeworkDir/AA-I-Homework-15.aspx

1 2 3 4

Find the Laurent series for

$$f(z) = \frac{\sin(2z)}{z^2}$$

at $z_0 = 0$

$a_{-1} =$

$a_0 =$

[Calculate and submit grade](#)

Online homework

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Untitled Page - Windows Internet Explorer

http://localhost:54348/Homework/HomeworkDir/Cal-III-Homework-4.aspx

Live Search

Untitled Page

Calculate the equation of plane that passess through the following points

A=(1,0,0)
B=(0,1,0)
C=(0,0,-2)

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = [\text{ } , \text{ } , \text{ }]$

Equation of plain (for example $x+y-z-2=0$)

=0

(symbolic formula)

Submit the answer

Done

Local intranet | Protected Mode: On

100%

Online visualization

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Lx	10	
Ly	10	
ax	30	
ay	30	
k	1	
q	100	
dt	0.05	
Tinitial	10	
Tmin	10	
Tmax	50	

Start calculations

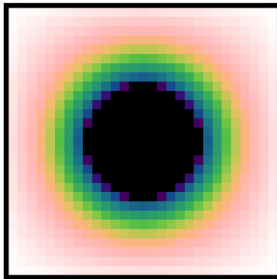


Figure: Solution of the heat transfer equation

Sample Problem

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- Find area of the parallelogram for $\bar{a} = [1, 2, 3]$, $\bar{b} = [3, 2, 1]$. Answer: $A = |\bar{a} \times \bar{b}| = 4\sqrt{6}$.
- How to input $4\sqrt{6}$ into the system in order to provide the answer?
- It is possible to use text description of the expression. For example:
 - $4 * \text{sqrt}(6)$
 - $4 * \text{Sqrt}[6]$
 - $4\text{sqrt}(6)$
 - $4\text{sqrt}6$
 - $4 \cdot \text{sq}6$
 - etc.

Parse Tree

Motivation

Online learning

How to create online assignments?

Typical problems in mathematical representation of the student's answers

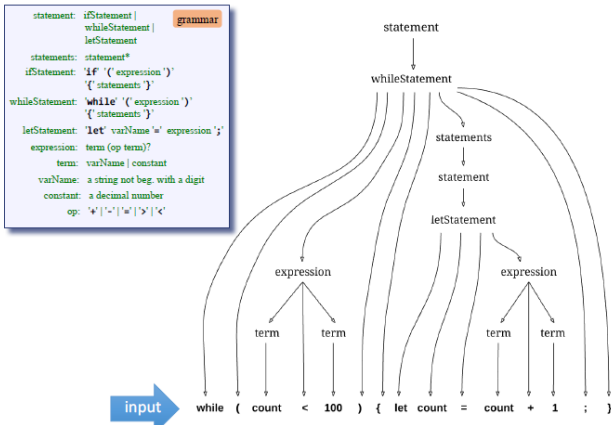
Arithmetic expressions with errors

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Conclusions

Parse tree



XML Parse Tree

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Parse tree

```
statement: ifStatement |  
          whileStatement |  
          letStatement  
statement: statement*  
ifStatement: 'if' '(' expression ')'  
            '{' statements '}'  
whileStatement: 'while' '(' expression ')'  
              '{' statements '}'  
letStatement: 'let' varName '=' expression ';'   
expression: term (op term)?  
            term: varName | constant  
            varName: a string not beg. with a digit  
            constant: a decimal number  
            op: '+' | '-' | '*' | '/' | '<' | '>'
```

grammar

```
<whileStatement>  
<keyword> while </keyword>  
<symbol> ( </symbol>  
<expression>  
  <term>  
    <identifier> count </identifier>  
  </term>  
  <symbol> < </symbol>  
  <term>  
    <IntConstant> 100 </IntConstant>  
  </term>  
</expression>  
<symbol> ) </symbol>  
<symbol> { </symbol>  
<statements>  
  <letStatement>  
    <keyword> let </keyword>  
    <identifier> count </identifier>  
    <symbol> = </symbol>  
    <expression>  
      <term> <identifier> count </identifier> </term>  
      <symbol> + </symbol>  
      <term> <IntConstant> 1 </IntConstant> </term>  
    </expression>  
    <symbol> ; </symbol>  
  </letStatement>  
</statements>  
<symbol> } </symbol>  
</whileStatement>
```

parser output

Same parse tree,
in XML

Grammar

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Lexical elements:	The Jack language includes five types of terminal elements (tokens):
keyword:	'class' 'constructor' 'function' 'method' 'field' 'static' 'var' 'int' 'char' 'boolean' 'void' 'true' 'false' 'null' 'this' 'let' 'do' 'if' 'else' 'while' 'return'
symbol:	'(' ')' '[' ']' '{' '}' ';' ':' ',' '.' '/' '*' '%' '^' '&' ' ' '<' '>' '=' '~'
integerConstant:	A decimal number in the range 0 .. 32767.
StringConstant:	"" A sequence of Unicode characters not including double quote or newline ""
identifier:	A sequence of letters, digits, and underscore ('_') not starting with a digit.
Program structure:	A Jack program is a collection of classes, each appearing in a separate file. The compilation unit is a class. A class is a sequence of tokens structured according to the following context free syntax:
class:	'class' className '{' classVarDec* subroutineDec* '}'
classVarDec:	('static' 'field') type varName (',' varName)* ';'
type:	'int' 'char' 'boolean' className
subroutineDec:	('constructor' 'function' 'method') ('void' type) subroutineName '(' parameterList ')' subroutineBody
parameterList:	((type varName) (',' type varName)*)?
subroutineBody:	'{' varDec* statements '}'
varDec:	'var' type varName (',' varName)* ';'
className:	identifier
subroutineName:	identifier
varName:	identifier
Statements:	
statements:	statement*
statement:	letStatement ifStatement whileStatement doStatement returnStatement
letStatement:	'let' varName '(' 'I' expression ')' '=' expression ';'
ifStatement:	'if' '(' expression ')' '{' statements '}' ('else' '{' statements '}')?
whileStatement:	'while' '(' expression ')' '{' statements '}'
doStatement:	'do' subroutineCall ';'
ReturnStatement:	'return' expression? ';'
Expressions:	
expression:	term (op term)*
term:	integerConstant stringConstant keywordConstant varName varName '(' expression ')' subroutineCall '(' expression ')' unaryOp term
subroutineCall:	subroutineName '(' expressionList ')' ((className varName) '.' subroutineName '(' expressionList ')')?
expressionList:	(expression (',' expression)*)?
op:	'+' '-' '*' '/' '%' '^' '&' ' ' '<' '>' '=' '~'
unaryOp:	'-' '~'
KeywordConstant:	'true' 'false' 'null' 'this'

Infix notation, Prefix notation, Postfix notation

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Conclusions

Different notation for arithmetic expressions.

- Infix notation $(5 + 6) \times 7$
- Prefix notation $x + 567$
- Postfix notation $756 + x$

Typical evaluation process of arithmetic expressions.

- $\text{InfixToPrefix}((5 + 6) \times 7) = x + 567$
- $\text{EvaluatePrefix}(x + 567) = 77$

Expression Tree

Motivation

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How to create
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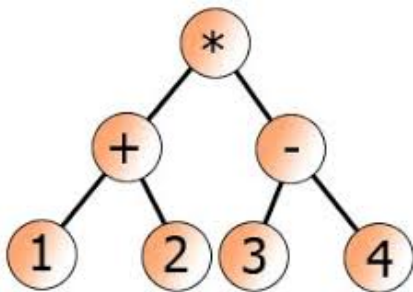
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$((1+2)*(3-4))$

How to evaluate mathematical expression Given as a String?

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Example

- Expression: `sqrt(2)*6`
- Value: $\sqrt{2} \cdot 6$

Example

- Expression: `sqrt2*6`
- Value: $\sqrt{2} \cdot 6$

Example

- Expression: `Sqrt[2]6`
- Value: $\sqrt{2} \cdot 6$

Example

- Expression: `SQRT[2]6`
- Value: $\sqrt{2} \cdot 6$

WebAssign question modes

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Mode	Description
Algebraic	Students submit a mathematical expression or equation that is evaluated algebraically.
Essay	Students submit an extended textual response. Scored manually.
File-Upload	Students submit a file. Scored manually.
Fill-in-the-Blank	Students submit a brief textual response.
Graphing	Students draw on a Cartesian coordinate plane.
Image Map	Students click a displayed image.
Java	Students use Java, Flash, or other applet to answer the question.
Matching	Students match items from two lists.
Multiple-Choice	Students select one response from a list.
Multiple-Select	Students select one or more responses from a list.
NumberLine	Students place or draw points, lines, segments, or rays.
Numerical	Students submit a numerical response which might include units or be checked for significant figures.
Poll	Used with other modes to create questions that collect information only. All responses are scored correct.
Symbolic	Students submit a mathematical expression (not an equation) that is evaluated by value substitution.

WebAssign questions from code (HTML, Perl etc.)

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The following code in Answer accepts two values between 20 and 100, and then requires your students to multiply them:

```
<EQN $A=userinput(20,100,50)>
```

```
<EQN $B=userinput(20,100,50)>
```

```
<EQN $SIMPLIFIED=1; $A * $B>
```

To require students to perform the computations instead of stating their response as a mathematical expression, set the `$SIMPLIFIED`, `$FRACTION`, or `$PROPERFRACTION` variables to 1 as you would for any other numerical question.

Turing complete programming languages in teaching

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Any Turing complete programming language (e.g. C,C++,Python, Java etc.) can express wide class of algorithms.

This gives teachers a tool to work interactively online on problems with practically arbitrary complexity.

To use this tool, it is necessary to represent a mathematical problem in a form of computer code and then the students by using webpages can interact with this code to check their progress. In the same way it is possible to work with research problems with arbitrary complexity.

Curry–Howard correspondence

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The Curry–Howard correspondence (also known as the Curry–Howard isomorphism or equivalence, or the proofs-as-programs and propositions- or formulae-as-types interpretation) is the direct relationship between computer programs and mathematical proofs.

Teacher/scientists can use this relation and create computer programs from mathematical formulations and mathematical formulations from computer programs.

Arithmetic expressions with errors

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$$1 * +)2)$$

$$1 * +2)$$

$$5 + ^2$$

$$1 * +(2$$

$$1 * +2$$

$$1 + ^2$$

$$3 + 2$$

Correct formula

$$1 + 2$$

$$\frac{1 + 2}{()}$$

$$1 + 2 + 0$$

$$\frac{1 + 2}{(1 + ())}$$

$$1 + 2 * 0$$

$$1 + 2/0$$

Correct formulation is surrounded with incorrect statements

Motivation

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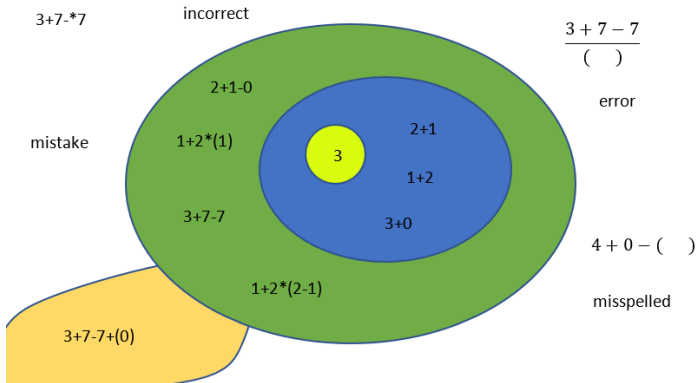
Typical problems in mathematical representation of the student's answers

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Mathematical fallacy - problems with syntax

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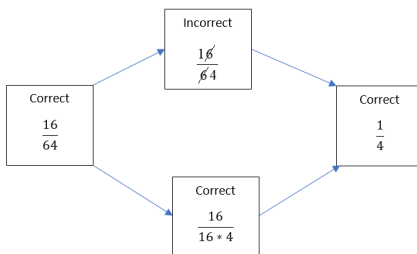
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$$\frac{d}{dx} \frac{1}{x} = \frac{\cancel{d}}{\cancel{d}x} \frac{1}{x} =$$
$$= -\frac{1}{x^2}.$$

Mathematical fallacy - problems with algebra

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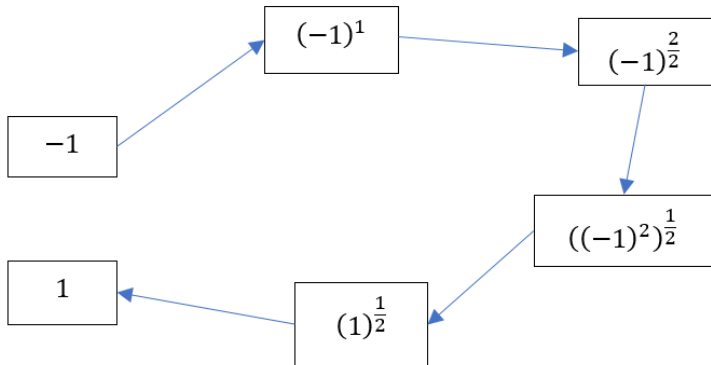
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Mathematical fallacy - problems with algebra

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$$\begin{aligned}-1 &= -1 \\ -1/1 &= -1/1 \\ -1/1 &= 1/-1 \\ \sqrt{-1/1} &= \sqrt{1/-1} \\ i/1 &= 1/i \\ i &= 1/i \\ i^2 &= 1 \\ -1 &= 1\end{aligned}$$

Problems with visual learning

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Example

$$\lim_{x \rightarrow 8^+} \frac{1}{x - 8} = \infty$$

Conclusion

$$\lim_{x \rightarrow 5^+} \frac{1}{x - 5} =$$

Problems with visual learning

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Example

$$\lim_{x \rightarrow 8^+} \frac{1}{x - 8} = \infty$$

Conclusion

$$\lim_{x \rightarrow 5^+} \frac{1}{x - 5} = 5$$

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$$a \times b = ab$$

Conclusion

$$2 \times 2 = ?$$

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Example

$$a \times b = ab$$

Conclusion

$$2 \times 2 = 22(\textit{incorrect})$$

$$2 \times 2 = 4(\textit{correct})$$

Problems with visual learning

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Example

$$a \times b = ab$$

Conclusion

$$2 \times 2 = 22(\textit{incorrect})$$

$$2 \times 2 = 4(\textit{correct})$$

String concatenation

Example

$$'a' + 'b' = 'ab'$$

Conclusion

$$'2' + '2' = '?'$$

Problems with visual learning

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Example

$$a \times b = ab$$

Conclusion

$$2 \times 2 = 22(\textit{incorrect})$$

$$2 \times 2 = 4(\textit{correct})$$

String concatenation

Example

$$'a' + 'b' = 'ab'$$

Conclusion

$$'2' + '2' = '4'(\textit{incorrect})$$

$$'2' + '2' = '22'(\textit{correct})$$

Problems with classification

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Is this true that this is a cat?



Answer:

Problems with classification

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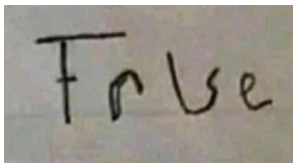
Learning

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Is this true that this is a cat?



Answer:

The word "False" is written in a casual, handwritten style using black ink on a light-colored, slightly textured background.

Error correction - example

Motivation

Online learning

How to create online assignments?

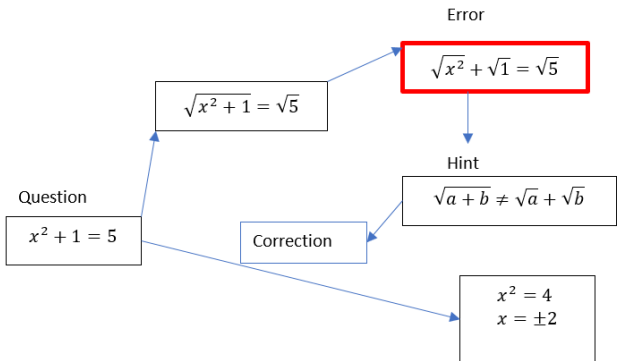
Typical problems in mathematical representation of the student's answers

Arithmetic expressions with errors

Multi-valued logic

Learning

Conclusions



Error correction - generalization

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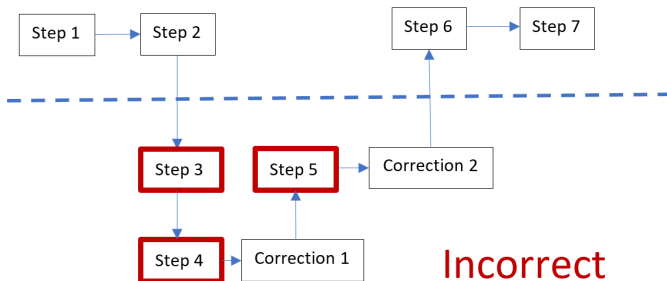
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Correct



Arithmetic operators with multiple form

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Input expression with a syntax error

$$2 * (3 + */2)$$

Arithmetic operators with multiple form

$$\{+, *, /\}$$

Possible multiple form of the expression

$$2 * (3\{+, *, /\}2) = \{2 * (3 + 2), 2 * (3 * 2), 2 * (3/2)\}$$

Missing parts of the expression

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Input expression with a syntax error

$$2 * (3 + 2$$

Input expression with correction

$$2 * (3 + 2)$$

Missing parts of the expression - values

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Input expression with a syntax error

$$2 * (3+)$$

Input expression with correction

$$2 * (3 + \{x, y, f(x), g(x, y)\})$$

$$\{2 * (3 + x), 2 * (3 + y), 2 * (3 + f(x)), 2 * (3 + g(x, y))\}$$

where x, y are some real/integer/rational values.

$f(x), g(x, y)$ are some functions/expressions.

Applications of multi-valued logic for ordering of errors in arithmetic expressions

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Input expression with a syntax error

$$2 * (3 + */2)$$

Correct expression

$$2 * (3 + 2)$$

Partial ordering relation \leq between “more correct” and “less correct” expressions

$$2 * (3 + */2) \leq 2 * (3 + *2) \leq 2 * (3 + 2)$$

Some expressions have similar level of “correctness”.

$$2 * (3 + *2) \approx 2 * (3 + /2) \approx 2 * (3 * /2)$$

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Correctness of some expressions cannot be compared

$$2 * (3 + 2) \not\equiv 2 * (3 + *2)$$

In the expression $2 * (3 + 2)$ there is a problem with parentheses.
In the expression $2 * (3 + *2)$ there is a problem with mathematical operator.

$$2*)3 + *2 \not\equiv 1 + 2*$$

'Hesse diagram' for syntax errors

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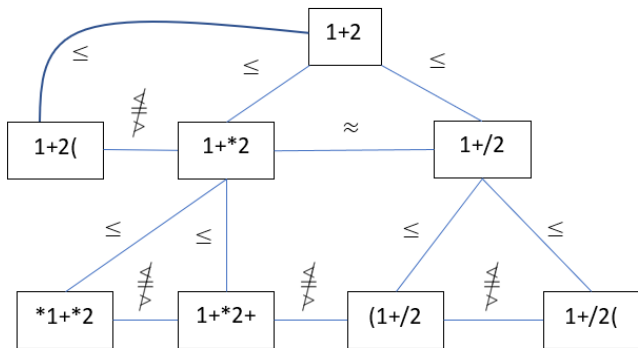
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Learning as minimization of functional

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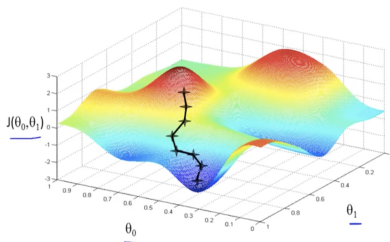
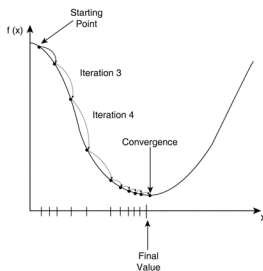
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Learning as sequence of updating

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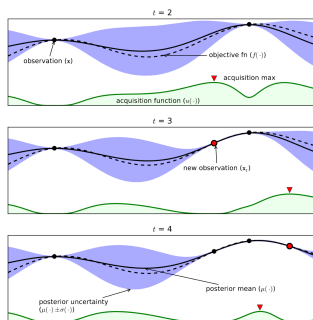
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Recursive Bayesian estimation

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

Learning as sequence of explanations, trials, and corrections

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The Learning Curve

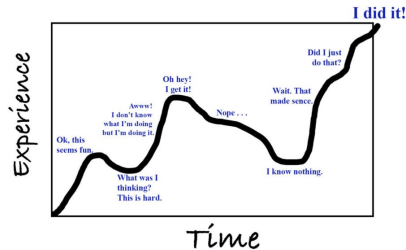


Figure: Learning process is represented as a curve

Learning described in this presentation is modeled as a sequence of explanations, trials, and corrections.

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- Turing complete programming languages give teachers a tool to work interactively online on problems with practically arbitrary complexity.
- Syntax and grammar analysis of the mathematical statements can improve online learning systems.
- Arithmetic expressions with errors can have possibly many different corrections and meanings.
- Some corrections can be represented as set operations.
- Multi-valued logic can be used to study various stages of correctness.
- Presented approach can be use as a mathematical foundation for “partial credits”.
- Learning can be viewed as a sequence of explanations, trials, and corrections.

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Thank You