System of Equations with Random Set Parameters

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Random variable

\[ X : \Omega \ni \omega \rightarrow X(\omega) \in R \]
Probability density function
Random set

\[ X(\omega) = \left[ X(\omega), \bar{X}(\omega) \right] \]

\[ p_i = P_\Omega \{ \omega_i \} \]

\[ \sum_{\omega_i \in \Omega} P_\Omega \{ \omega_i \} = \sum_i p_i = 1 \]
Upper and lower probability

\[ Pl(A) = \sum_{i: A \cap X_i \neq \emptyset} p_i \]

\[ Bel(A) = \sum_{i: X_i \subseteq A} p_i \]
The theory of clouds

\[ p_i = P_\Omega \{ X \in X_i \} \]
Cumulative distribution function

\begin{align*}
F(x) &= P_\Omega \{ X \leq x \} \\
\end{align*}

\[ P([a,b]) = F(b) - F(a) \]
Cumulative distribution function in the case of random sets

\[ F(x) = P_\Omega \{ \overline{X} \leq x \} \]

\[ \overline{F}(x) = P_\Omega \{ \underline{X} \leq x \} \]
Example – upper probability

<table>
<thead>
<tr>
<th>Upper</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
Upper probability

\[ Pl([a, b]) = \bar{F}(b) - \bar{F}(a) \]
Functions of random variables

\[ y = g(x) \]

\[ f_X(x) - \text{PDF of the random variable } X. \]

\[ f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d g^{-1}(y)}{d x} \right| \]

It is necessary to describe random sets by using Monte Carlo simulations.
1) Generate the interval

\[ X_i = [\underline{X}_i, \bar{X}_i] \]

2) Calculate

\[ Y_i = [\underline{Y}_i, \bar{Y}_i] = g(X_i) \]

3) Update \( E_i(y), \bar{F}_i(y) \).

4) Return to 1.
Numerical example


Equation with the random set parameters $x^2 - a = 0$ where $a_l = 9 + N(0, 1)$ $a_u = a_l + 1$
System of equations

\[
\begin{align*}
ax + by &= a^2 + b^2 \\
ax - by &= a^2 - b^2
\end{align*}
\]
Conclusions

- Using special version of Monte Carlo simulations it is possible to solve system of equations with the random-sets parameters.

- Example solution are available on-line and they are implemented in special java applets.