HYBRID INTERVAL METHOD FOR GLOBAL OPTIMIZATION OF ENGINEERING STRUCTURES

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Abstract:

The problem of optimal design consists in finding the optimum parameters according to a prescribed optimality criterion. Existing optimization methods usually aren't reliable or can't use the nondifferentiable, not continuous objective functions or constraints. An interval global optimization method is: very stabile and robust and universally applicable. The interval algorithm guarantees that all stationary global solutions have been found. Unfortunately application of this algorithm is sometimes a very time consuming task. The best local optimization methods are the gradient methods. In this paper a hybrid gradient-interval global optimisation method is presented. This method has the best features of both methods i.e. fast local and reliable global convergence. In this paper this algorithm was applied to optimization of truss structures. Examples of optimization of truss structures with uncertain parameters was also presented.

1. INTRODUCTION

Algorithms for solving global minimization problems can be classified into heuristic methods that find the global minimum only with high probability, and methods that guarantee to find a global minimum with accuracy. An important class belonging to the former type are the stochastic methods. A number of techniques like simulated annealing and genetic algorithms use analogies to physics and biology to approach the global optimum. The most important class of methods of the second type are branch and bound methods [2]. They derive their origin from combinatorial optimization, where global optima are also wanted but the variables are discrete and take a few values only. Branch and bound methods guarantee to find a global minimum with a desired accuracy after a predictable (though often exponential) number of steps. The basic idea is that the configuration space is split recursively by branching into smaller and smaller parts. This is not done uniformly, instead some parts are preferred and others are eliminated. The details depend on bounding procedures. Lower bounds on the objective allow to eliminate large portions of the configuration space early in

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the computation so that only a (usually small) part of the branching tree has to be generated and processed. The lower bounds may be obtained by using techniques of interval analysis [3], or other method. The interval global optimization method is: very stabile, robust and universally applicable. The interval algorithm guarantees that all stationary global solutions have been found. Unfortunately this algorithm is sometimes very time consuming [4].

Local optimization methods are usually much faster. In this paper an optimization method based on the sensitivity analysis was applied.

A hybrid algorithm is constructed in the following way. First local minimum is found using a local optimization method, then we check if this minimum is global using the algorithm of interval global optimization.

2. A LOCAL OPTIMIZATION METHOD

The mathematical modelling of structural design optimization problem is as follows:

$$\begin{cases} min\ J(\mathbf{x}) \\ g_j(\mathbf{x}) \geq 0 \quad \text{for} \quad j = 1,\ldots,m_{in} \\ p_i(\mathbf{x}) = 0 \quad \text{for} \quad i = 1,\ldots,m_{eq} \end{cases} \tag{1}$$

where $J : \mathrm{R}^{\mathrm{n}} \to \mathrm{R}$ is the objective function, n is the number of design variables x_i, m_{in} is the number of inequality constraints, m_{eq} is a number of equality conditions.

The objective function J is equal to the weight of the truss

$$J = \sum_{i=1}^{ne} \gamma \cdot A_i \cdot L_i \tag{2}$$

The inequality constraints are defined in the following way:

$$g_j(x) = \sigma_0 - |\sigma_i| \geq 0 \quad \text{or} \quad g_j(x) = |\sigma_i| A_i - \frac{\pi^2 E J_i}{L_i^2} \geq 0 \tag{3}$$

where σ_0 is the allowable stress, σ_i is the stress in the i-th bar, L_i is the length of the i-th bar, E is the Young's modulus, J_i is the moment of inertia of the cross-section of the i-th bar, γ is the specific gravity of the material. The equality constraints are given in the form of the equilibrium equations:

$$\mathbf{K}(\mathbf{x})\mathbf{q} = \mathbf{Q} \tag{4}$$

where **K** is the global stiffness matrix, **q** is the displacement vector, **Q** is the force vector.

The procedure presented here is based on the first order gradient-based method (e.g. [1]), but can be replaced by any local optimization algorithm.

The steps of presented procedure are the following:

1. Initial configuration $i = 0, \mathbf{x} = \mathbf{x}^0$.
2. Evaluate $J(\mathbf{x}^i)$ and $g_j(\mathbf{x}^i)$ for j=1,…, m_{in}.
3. Identify violated or active constraints.
4. Calculate the gradient using the sensitivity analysis methods.
5. Determine the next search direction $\mathbf{s}^i$.
6. Perform one-dimensional search to find α.
7. Set $\mathbf{x}^{i+1} = \mathbf{x}^i + \alpha \cdot \mathbf{s}^i$.
8. Check for convergence to the optimum. If convergence criteria are satisfied, exit, otherwise set $i = i + 1$, and return to step 2.

3. THE INTERVAL GLOBAL OPTIMIZATION

The interval global optimization method is based on the properties of interval arithmetic (3). If the following inequality holds

$$\hat{f}([\mathbf{x}_1])^+ < \hat{f}([\mathbf{x}_2])^- \qquad (5)$$

where $\left[\hat{f}([\mathbf{x}])^-, \hat{f}([\mathbf{x}])^+\right] = hull\ \hat{f}([\mathbf{x}])$, $[\mathbf{x}_1],[\mathbf{x}_2] \in \mathrm{IR}^n$, $f : \mathrm{R}^n \to \mathrm{R}$, then the global minimum is not in the interval $[\mathbf{x}_2]$ and can be omitted in future calculations. Because $\hat{f}([\mathbf{x}_1])^+ < \hat{f}([\mathbf{x}_2])^-$ then from the fundamental property of interval arithmetic, it follows that

$$\forall \hat{\mathbf{x}} \in [\mathbf{x}_1] \forall \breve{\mathbf{x}} \in [\mathbf{x}_2]\ f(\hat{\mathbf{x}}) < f(\breve{\mathbf{x}}), \qquad (6)$$

hence the global minimum of the function *f* is not in interval $[\mathbf{x}_2]$ and $[\mathbf{x}_2]$ can be omitted in future calculations. IR is a set of all closed intervals [3], $\hat{f} : \mathrm{IR}^n \to \mathrm{R}$ is the natural interval extension of the function *f* [3].

Let $[\mathbf{x}] \in \mathrm{IR}^n$ be an initial interval. The basic algorithm is as follows [2]:

Step 0 Set $[\mathbf{y}] = [\mathbf{x}]$ and $y = \hat{f}([\mathbf{x}])^-$. Initiate the list L=(($[\mathbf{y}]$,*y*)) and the cut-off level z = $\hat{f}([\mathbf{x}])^+$.

Step 1 Choose a coordinate direction $k \in \{1,2,\ldots,n\}$.

Step 2 Bisect **y** in direction k: $[\mathbf{y}] = [\mathbf{v}_1] \cup [\mathbf{v}_2]$.

Step 3 Calculate $\hat{f}([\mathbf{v}_1])$ and $\hat{f}([\mathbf{v}_2])$ and set $\mathrm{v}_i = \hat{f}([\mathbf{v}_i])^-$ for i=1,2 and $z = \min\{z,\ \hat{f}([\mathbf{v}_1])^+,\ \hat{f}([\mathbf{v}_2])^+\}$.

Step 4 Remove ($[\mathbf{y}]$, *y*) from the list L.

Step 5 Cutoff test: discard the pair $([\mathbf{v}_i], \mathrm{v}_i)$ if v_i , z (where i=1, 2).

Step 6 Add any remaining pair(s) to the list L. If the list becomes empty then STOP.

Step 7 Denote the pair with the smallest second element by $([\mathbf{y}], y)$.

Step 8 If the width of $\hat{f}([\mathbf{y}])$ is less than ε, then print $\hat{f}([\mathbf{y}])$ and $[\mathbf{y}]$, STOP.

Step 9 Go to step 1.

4. A NUMERICAL EXAMPLE

Let us consider the truss structure which is shown in Fig 1.

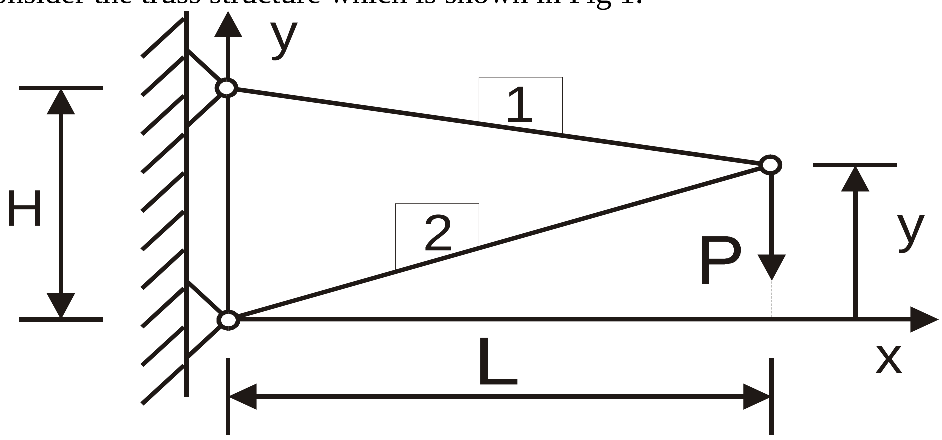

Fig 1

In calculation, we assume that L=H=1 [m], σ_0=190 [MPa], P=10 [kN], $[x_w]=[0,2]$ [m], $[y_w]=[0, 1]$ [m]. The numerical results are shown in the table 1.

Table 1

P [kN]	Local optimization			Interval global optimization		
	y [m]	A_1 [m^2]	A_2 [m^2]	y [m]	A_1 [m^2]	A_2 [m^2]
1	0.4995	$5.88\cdot 10^{-6}$	$5.88\cdot 10^{-6}$	[0.497,0.505]	$5.88\cdot 10^{-6}$	$5.88\cdot 10^{-6}$

The results were calculated in 12 iterations of the interval algorithm with monotonicity test [2]. From table 1 and properties of the interval global optimization method follow that presented problem has only one global minimum.

5. CONCLUSIONS

In this paper a new method of global optimization of engineering structures was presented. This method is much more effective than pure interval global optimization method and can also prove existence and unique of the global minimum in the nonlinear global optimization problems. More complicated examples of applications will be presented on the conference.

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