## Applications of Self-Adaptive Computational Methods in Online Learning

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### Outline



- 2 Automatically Generated Examples
- Self Adaptive Computational Methods





# **Online Learning**

#### Online Learning

Automatically Generated Examples

Self Adaptive Computational Methods

Motivation

Conclusions

### **TA Homework**

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Username:	ta						
First Name:	Andrew						
Last Name:	Pownuk						
Group:	2018-Spring-MATH-4329-CRN-24656						
	Logoff						

Number of homework: 12

Create new Homework

homework-description	Homework 13
homework-id	Homework-13
link	homework/Homework-13.aspx
tries-max	10

#### Online Learning

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- Sharing the lecture notes.
- Interactive platform for doing online homework.

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- Automated system for checking attendance.
- Integrated response system.
- Grades management system.
- Interactive projects.

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First Name.	Andrew			
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	2/13/2018	quiz-8		2/13/2018	quiz-9		2/13/2018	quiz- 10		2/13/2018
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### Is it possible to create automated thinker?



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## Sample assignment (simplify the expression)



$$0$$

$$1-1$$

$$(1-1)*(1-1)$$

$$(1-1)*(1-1)*5$$

$$(1-1)*(1-1)*5*\frac{7}{2}$$

$$(1-1)*\left(\frac{3}{3}-1\right)*5*\frac{7}{2}$$

$$-\frac{99}{99} \times \left(\frac{3+(1-1)*\left(\frac{3}{3}-1\right)*5*\frac{7}{2}}{3}-1\right)*5*\frac{7}{2}$$

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### Sample assignment (find integral)



7x + 5  $7\left(\frac{x^2}{2}\right)' + 5(x)'$   $\left(7\frac{x^2}{2} + 5x + C\right)'$   $\int (7x + 5)dx = 7\frac{x^2}{2} + 5x + C$ 

## Sample assignment (find derivative)

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$$((3x + 1) * (5x + 7))'$$
  
 $(3x + 1)' * (5x + 7) + (3x + 1) * (5x + 7)'$   
 $3 * (5x + 7) + (3x + 1) * 5$ 

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### Sample assignment (calculate the limit)



$$\lim_{x \to \infty} \frac{5x^2 + 2x + 1}{x^2 + 1}$$
$$\lim_{x \to \infty} \frac{\frac{5x^2 + 2x + 1}{x^2}}{\frac{x^2 + 1}{x^2}}$$
$$\lim_{x \to \infty} \frac{5 + \frac{2}{x} + \frac{1}{x^2}}{1 + \frac{2}{x^2}}$$
5

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### Set theory

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 $A \cup B = B \cup A$  $A \cap A^{C} = \emptyset$  $(A \cup B) \cap (B \cup A)^{C} = B \cap B^{C}$  $((A \cup B) \cap (B \cup A)^{C}) \cap ((A \cup B) \cap (B \cup A)^{C})^{C} = \emptyset$ etc.

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### Probability theory



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$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A|B)P(B) = P(A \cap B)$$
$$P(A \cup B) = P(A) + P(B) - P(A|B)P(B)$$

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etc.

### Analysis

#### Online Learning

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$$\frac{d}{dx}\left(EA\frac{du}{dx}\right) + n = 0, u(0) = 0, u(L) = 0$$
$$\int_{0}^{L} \frac{d}{dx}\left(EA\frac{du}{dx}\right)vdx + \int_{0}^{L} nvdx = \int_{0}^{L} 0vdx, u(0) = 0, u(L) = 0$$
$$\int_{0}^{L} u\frac{dv}{dx}dx = \int_{0}^{L} \frac{du}{dx}vdx + u(0)v(L) - u(L)v(L)$$
$$\int_{0}^{L} \frac{d}{dx}\left(EA\frac{du}{dx}\right)vdx = \int_{0}^{L} EA\frac{du}{dx}\frac{dv}{dx}dx + EA\frac{du(0)}{dx}v(0) - EA\frac{du(L)}{dx}v$$

etc.



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Fundamental theorem of Galois theory. Number of automorphisms equals the degree of the extension.

|Gal(K/F)| = [K:F]

Field extension K/F. Degree of the extension [K : F]. Number of automorphisms in the Galois extension |Gal(K/F)|. For example, if we know that  $6 = [K : F] = \left[\mathbb{Q}\left(\sqrt[3]{2}, \sqrt{-3}\right) : \mathbb{Q}\right]$ , then we know instantly that  $6 = |Aut(K/F)| = |Aut\left(\mathbb{Q}\left(\sqrt[3]{2}, \sqrt{-3}\right)/\mathbb{Q}\right)|$  etc.

### Calculus I

#### Online Learning

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Lagrange Theorem.

- Function f(x) is continuous in the interval [a, b]
- Function f(x) is differentiable in the interval (a, b)

then exists  $c \in (a, b)$  such that  $\frac{f(b)-f(a)}{b-a} = f'(c)$ 

- Function f(x)g(x) is continuous in the interval [a, b]
- Function f(x)h(x) is differentiable in the interval (a, b)

then exists  $c \in (a, b)$  such that  $\frac{f(b)g(b)-f(a)h(a)}{b-a} = f'(c)g(c) + f(c)g'(c)$ 

### Programming



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```
int f(int x)
{
         return x*x;
}
f(f(x))
int ff(int x)
ł
         return (x*x)*(x*x);
}
```

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### Natural Language

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Input information: I like Las Cruces.

Output information: I like a city 40 miles from El Paso.

Output information:

I like a place with a lot of houses and roads 40 miles from El Paso.

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### Self Adaptive Computational Methods

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Product rule (input information)

$$(f \ast g)' = f' \ast g + f \ast g'$$

After calculations (new theorem created automatically)

$$(f * g * h)' = f' * g * h + f * g' * h + f * g * h'$$

New theorem can be used in exactly the same way like the original theorem.

### System of linear equations

- Online Learning
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Regular student has a lot of problems with solution of a system of equations with more than 5 equations.

By using computational methods it is possible to solve millions of equations.

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### Integration/differentiation

#### Online Learning

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Regular student has a lot of problems finding the solutions of the typical problems from the calculus I and II textbook.

By using computational algebra system it is possible to solve millions of integrals in reasonable time.

### AlphaGo

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AlphaGo's 4-1 victory in Seoul, South Korea, in March 2016 was watched by over 200 million people worldwide. It was a landmark achievement that experts agreed was a decade ahead of its time, and earned AlphaGo a 9 dan professional ranking (the highest certification) - the first time a computer Go player had ever received the accolade.

### Google's AlphaZero

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December 6, 2017 Google's AlphaZero defeated Stockfish in 100-game match. Alpha Zero won the match with 28 wins,72 draws and 0 lose.

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### Conclusions

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- By using presented methodology it is possible to create complex educational examples in many areas of mathematics as well as in other areas of science and engineering.
- In a few minutes it is possible to create thousands pages with typical examples that can be used in education.
- By using self adaptive computational methods it is possible to automatically generate new mathematical theorems complectly independently from human interactions.