Solution of Algebraic Equations by Using Autonomous Computational Methods

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Outline

Checkers (Game Theory)

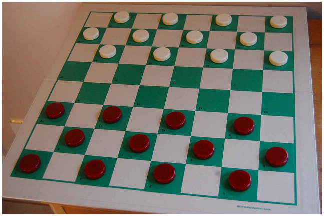


Figure: Game tree

Arthur Samuel in 1959 applied alphabeta pruning in order to create a program that learned how to play checkers. Alphabeta pruning is a search algorithm that seeks to decrease the number of nodes that are evaluated by the minimax algorithm in its search tree.

Game Theory (tic-tac-toe)

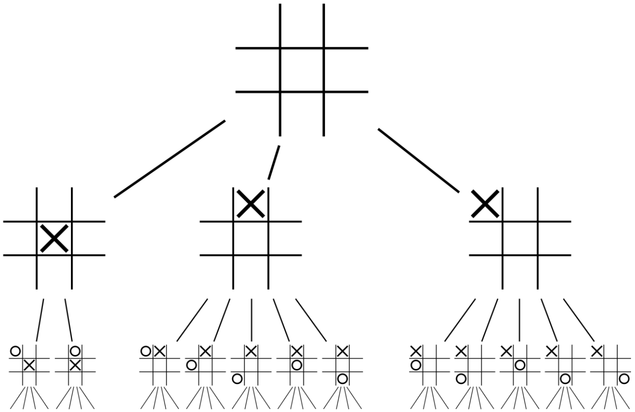


Figure: Game tree

Game Theory (GO)

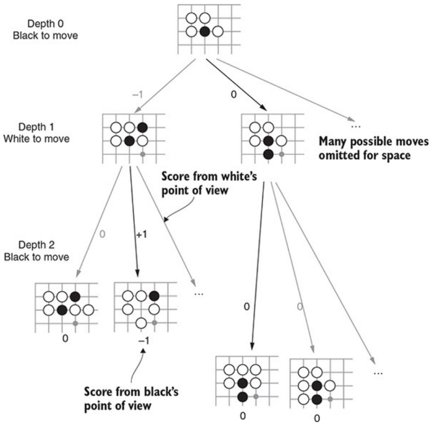


Figure: Game tree

Game Theory (Winning Strategy)

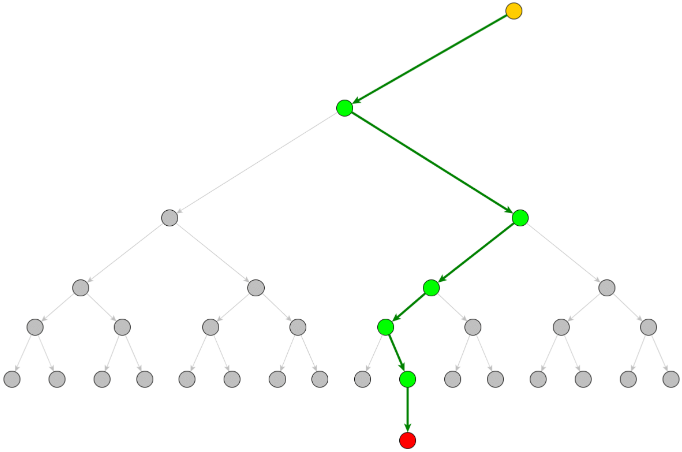


Figure: Game tree

Basic Mathematical Operations

Addition

$$1 + 1 = 2$$

$$1 + 0 = 1$$

$$2 + (-1) = 1$$

Multiplication

$$1 * 0 = 0$$

$$i * i = -1$$

Algebra

$$a + b = b + a$$

$$a(b + c) = ab + ac$$

etc.

Self Adaptive Computational Methods

Product rule (input information)

$$(f * g)' = f' * g + f * g'$$

After calculations (new theorem created automatically)

$$((f * g) * h)' = (f * g)' * h + (f * g) * h'$$

$$((f * g) * h)' = (f' * g + f * g') * h + (f * g) * h'$$

$$(f * g * h)' = (f' * g) * h + (f * g') * h + (f * g) * h'$$

New theorem can be used in exactly the same way like the original theorem.

$$(f * g * h)' = f' * g * h + f * g' * h + f * g * h'$$

Lagrange Theorem.

- Function $f(x)$ is continuous in the interval $[a, b]$
- Function $f(x)$ is differentiable in the interval (a, b)

then exists $c \in (a, b)$ such that $\frac{f(b)-f(a)}{b-a} = f'(c)$

- Function $f(x)g(x)$ is continuous in the interval $[a, b]$
- Function $f(x)g(x)$ is differentiable in the interval (a, b)

then exists $c \in (a, b)$ such that
$\frac{f(b)g(b)-f(a)h(a)}{b-a} = f'(c)g(c) + f(c)g'(c)$

Differentiation

Input information

- $(f(x)+g(x))' = f'(x)+g'(x)$
- $(f(x)-g(x))' = f'(x)-g'(x)$
- $(f(x)g(x))' = f'(x)g(x)+f(x)g'(x)$
- $(f(x)/g(x))' = \frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$
- $(f(g(x)))' = f'(g(x))g'(x)$
- arithmetic operations

Differentiation

Automatically generate formulas out of:

- List of arithmetical operations
- List of elementary functions and constants
- Information about the numer of arithmetical operations and functions compositions

Differentiation (Latex source)

```
\begin{equation}\cos\left(x\right)+x+2+\sin\left(2\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(\sin\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(\cos\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(2\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(\sin\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(\cos\left(x\right)\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+2\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+x\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+2+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+x+x\end{equation}
\begin{equation}\cos\left(x\right)+x+2+x+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+x+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(x\right)+\sin\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\sin\left(x\right)+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+2+\cos\left(x\right)+\cos\left(x\right)\end{equation}
\begin{equation}\cos\left(x\right)+x+x+x\end{equation}
```

Differentiation (step 1)

$$\cos(x) + (\cos(x) + \cos(2)) \tag{765}$$

$$\cos(x) + (\cos(x) + \cos(x)) \tag{766}$$

$$\cos(x) + (\cos(x) + \cos(\sin(x))) \tag{767}$$

$$\cos(x) + (\cos(x) + \cos(\cos(x))) \tag{768}$$

$$\cos(x) + (\cos(x) + (2 + 2)) \tag{769}$$

$$\cos(x) + (\cos(x) + (2 + x)) \tag{770}$$

$$\cos(x) + (\cos(x) + (2 + \sin(x))) \tag{771}$$

$$\cos(x) + (\cos(x) + (2 + \cos(x))) \tag{772}$$

$$\cos(x) + (\cos(x) + (x + x)) \tag{773}$$

Differentiation (step 2)

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}\left(\cos(x) + \cos(2)\right) \tag{765}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}\left(\cos(x) + \cos(x)\right) \tag{766}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}\left(\cos(x) + \cos(\sin(x))\right) \tag{767}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}\left(\cos(x) + \cos(\cos(x))\right) \tag{768}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}\left(\cos(x) + (2 + 2)\right) \tag{769}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}\left(\cos(x) + (2 + x)\right) \tag{770}$$

$$\frac{d}{dx}\cos(x) + \frac{d}{dx}\left(\cos(x) + (2 + \sin(x))\right) \tag{771}$$

Differentiation (step 3)

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\sin(x)\right) \tag{762}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\sin(\sin(x))\right) \tag{763}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\sin(\cos(x))\right) \tag{764}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\cos(2)\right) \tag{765}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\cos(x)\right) \tag{766}$$

$$(-1)\cdot\sin(x)+\left(\frac{d}{dx}\cos(x)+\frac{d}{dx}\cos(\sin(x))\right) \tag{767}$$

Differentiation (step 4)

$$(-1)\cdot\sin(x) + \left((-1)\cdot\sin(x) + \left(\frac{d}{du}\cos(u)\right)\Bigg|_{u=\sin(x)} \cdot \frac{d}{dx}\sin(x)\right) \tag{767}$$

$$(-1)\cdot\sin(x) + \left((-1)\cdot\sin(x) + \left(\frac{d}{du}\cos(u)\right)\Bigg|_{u=\cos(x)} \cdot \frac{d}{dx}\cos(x)\right) \tag{768}$$

$$(-1)\cdot\sin(x) + \left((-1)\cdot\sin(x) + \left(\frac{d}{dx}2 + \frac{d}{dx}2\right)\right) \tag{769}$$

$$(-1)\cdot\sin(x) + \left((-1)\cdot\sin(x) + \left(\frac{d}{dx}2 + \frac{d}{dx}x\right)\right) \tag{770}$$

$$(-1)\cdot\sin(x) + \left((-1)\cdot\sin(x) + \left(\frac{d}{dx}2 + \frac{d}{dx}\sin(x)\right)\right) \tag{771}$$

$$(-1)\cdot\sin(x) + \left((-1)\cdot\sin(x) + \left(\frac{d}{dx}2 + \frac{d}{dx}\cos(x)\right)\right) \tag{772}$$

$$(-1)\cdot\sin(x) + \left((-1)\cdot\sin(x) + \left(\frac{d}{dx}x + \frac{d}{dx}x\right)\right) \tag{773}$$

Differentiation (step 5)

$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+\cos(\sin(x))\cdot\cos(x)) \tag{763}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+\cos(\cos(x))\cdot(-1)\cdot\sin(x)) \tag{764}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+0\cdot 0) \tag{765}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(-1)\cdot\sin(x)) \tag{766}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(-1)\cdot\sin(\sin(x))\cdot\cos(x)) \tag{767}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(-1)\cdot\sin(\cos(x))\cdot(-1)\cdot\sin(x)) \tag{768}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+0)) \tag{769}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+1)) \tag{770}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+\cos(x))) \tag{771}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(0+(-1)\cdot\sin(x))) \tag{772}$$
$$(-1)\cdot\sin(x)+((-1)\cdot\sin(x)+(1+1)) \tag{773}$$

Set theory

$$A \cup B = B \cup A$$
$$A \cap A^C = \emptyset$$
$$(A \cup B) \cap (B \cup A)^C = B \cap B^C$$
$$((A \cup B) \cap (B \cup A)^C) \cap ((A \cup B) \cap (B \cup A)^C)^C = \emptyset$$
$$etc.$$

Probability theory

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A|B)P(B) = P(A \cap B)$$
$$P(A \cup B) = P(A) + P(B) - P(A|B)P(B)$$

etc.

Analysis

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right)+n=0, u(0)=0, u(L)=0$$

$$\int_0^L \frac{d}{dx}\left(EA\frac{du}{dx}\right)vdx+\int_0^L nvdx=\int_0^L 0vdx, u(0)=0, u(L)=0$$

$$\int_0^L u\frac{dv}{dx}dx=\int_0^L \frac{du}{dx}vdx+u(0)v(L)-u(L)v(L)$$

$$\int_0^L \frac{d}{dx}\left(EA\frac{du}{dx}\right)vdx=\int_0^L EA\frac{du}{dx}\frac{dv}{dx}dx+EA\frac{du(0)}{dx}v(0)-EA\frac{du(L)}{dx}v($$

etc.

Algebra

Fundamental theorem of Galois theory.
Number of automorphisms equals the degree of the extension.

$$|Gal\,(K/F)| = [K:F]$$

Field extension K/F.
Degree of the extension $[K:F]$.
Number of automorphisms in the Galois extension $|Gal(K/F)|$.
For example, if we know that
$6 = [K:F] = \left[\mathbb{Q}\left(\sqrt[3]{2}, \sqrt{-3}\right) : \mathbb{Q}\right]$, then we know instantly
that $6 = |Aut\,(K/F)| = \left|Aut\left(\mathbb{Q}\left(\sqrt[3]{2}, \sqrt{-3}\right)/\mathbb{Q}\right)\right|$ etc.

Automated Programming


```
int f(int x)
{
        return x*x;
}
```

---

f(f(x))

```
int ff(int x)
{
        return (x*x)*(x*x);
}
```

Group Axioms

A group is a set (mathematics), G together with an Binary operation $''\cdot''$ (called the group law of G) that combines any two element elements a and b to form another element, denoted $a \cdot b$. To qualify as a group, the set and operation, $(G, \cdot)$, must satisfy four requirements known as the group axioms:

1. Closure: For all $a, b \in G$, the result of the operation, $a \cdot b$, is also in G.
2. Associativity: For all a, b and c in G, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
3. Identity element: There exists an element e in G such that, for every element a in G, the equation $1 = e \cdot a = a \cdot e = a$ holds. Such an element is unique.
4. Inverse element: For each $a \in G$, there exists an element $b \in G$, commonly denoted a^1 (or a, if the operation is denoted $+$, such that $a \cdot b = b \cdot a = e$, where e is the identity element.

Field Axioms

1. Addition is commutative: $x + y = y + x$, for all $x, y \in F$.
2. Addition is associative: $(x + y) + z = x + (y + z)$, for all $x, y, z \in F$.
3. Existence of additive identity: there is a unique element $0 \in F$ such that $x + 0 = x$, for all $x \in F$.
4. Existence of additive inverses: if $x \in F$, there is a unique element $x \in F$ such that $x + (x) = 0$.
5. Multiplication is commutative: $xy = yx$, for all $x, y \in F$.
6. Multiplication is associative: $(xy)z = x(yz)$, for all $x, y, z \in F$.
7. Existence of multliplicative identity: there is a unique element $1 \in F$ such that $1 \neq 0$ and $x1 = x$, for all $x \in F$.
8. Existence of multliplicative inverses: if $x \in F$ and $xneq0$, there is a unique element $(1/x) \in F$ such that $x(1/x) = 1$.
9. Distributivity: $x(y + z) = xy + xz$, for all $x, y, z \in F$.

Expression Evaluation

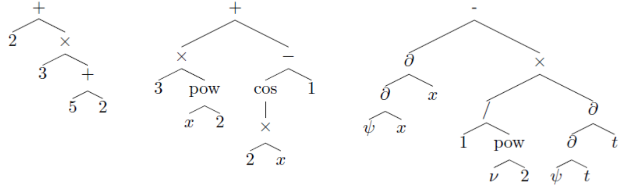
step-1
$(((1+2)+((1+2)-(3+4)))+(5+(6+7)))=((18-1)+0)$
step-2
$(((1+2)+((1+2)-(3+4)))+(5+(6+7)))=(17+0)$
step-3
$((3+((1+2)-(3+4)))+(5+(6+7)))=((18-1)+0)$
step-4
$(((1+2)+((1+2)-(3+4)))+(5+13))=((18-1)+0)$
step-5
$(((1+2)+(3-(3+4)))+(5+(6+7)))=((18-1)+0)$
step-6
$(((1+2)+((1+2)-7))+(5+(6+7)))=((18-1)+0)$
step-7
$(((1+2)+((1+2)-(3+4)))+(5+(6+7)))=17$
step-8
$((3+((1+2)-(3+4)))+(5+(6+7)))=(17+0)$
step-9
$(((1+2)+((1+2)-(3+4)))+(5+13))=(17+0)$
step-10
$(((1+2)+(3-(3+4)))+(5+(6+7)))=(17+0)$

Expression Evaluation

step-93
$((3+(-4))+18)=(17+0)$
step-94
$((-1)+(5+13))=(17+0)$
step-95
$((-1)+18)=((18-1)+0)$
step-96
$((3+(-4))+18)=17$
step-97
$((-1)+(5+13))=17$
step-98
$((-1)+18)=(17+0)$
step-99
$17=((18-1)+0)$
step-100
$((-1)+18)=17$
step-101
$17=(17+0)$
step-102
$17=17$

Expression evaluation is possible without specifying any explicit method for expression evaluation.

Computational Graph



COCONUT Project
COntinuous CONstraints - Updating the Technology.
https://www.mat.univie.ac.at/ neum/glopt/coconut/

Solution of Algebraic Equations

step-1
$x = (x + (2 + x))$
step-3
$x = (x + 2 + x)$
step-4
$x = (x + 2 + x)$
step-5
$x = x + 2 + x$
step-6
$x = x + 2 + x$
step-7
$x = 2 + 2 * x$
step-9
$x + (-1) * x + (-1) * x = 2$

Solution of Algebraic Equations

step-10
$x + (-2) * x = 2$
step-11
$(-1) * x = 2$
step-12
$x = (2/(-1))$
step-13
$x = (-2)$
step-14
$x = ((-2)/1)$

Automatically Generated Latex Report


```
step-1\\
$x=(x+(2+x))$
\newline
step-3\\
$x=(x+2+x)$
\newline
step-4\\
$x=(x+2+x)$
\newline
step-5\\
$x=x+2+x$
\newline
step-6\\
$x=x+2+x$
\newline
step-7\\
$x=2+2*x$
\newline
step-9\\
$x+(-1)*x+(-1)*x=2$
```

Automatically Generated PDF Reports

Hundred/thousand/unlimited number of examples:

51810	data-node-126.pdf
52690	data-node-127.pdf
52354	data-node-128.pdf
52785	data-node-130.pdf
52603	data-node-131.pdf
52512	data-node-132.pdf
52392	data-node-134.pdf
52565	data-node-135.pdf
53310	data-node-136.pdf
50225	data-node-14.pdf
52728	data-node-142.pdf
51848	data-node-143.pdf
52303	data-node-144.pdf
52666	data-node-146.pdf
52696	data-node-147.pdf
52484	data-node-148.pdf
50286	data-node-15.pdf
52539	data-node-150.pdf

http://andrew.pownuk.com/research/AlgebraicEquations/

Simplification of the Solution

step-1
$(4+6) = (x+x)$
step-3
$4+6 = (x+x)$
step-7
$4+6 = x+x$
step-11
$4+6 = 2*x$
step-15
$4+6+(-2)*x = 0$
step-21
$(-2)*x = 0+(-1)*4+(-1)*6$
step-24
$(-2)*x = (-10)$
step-25
$x = ((-10)/(-2))$
step-26
$x = 5$

Simplification of the Solution

- Finding optimal form of the computational process is one or the main goal of this project.
- Quality of the simplification depend on the amount of knowledge available in the system and processing power.
- For small problem the simplification can be found uniquely.
- For more complex problem simplification of the expressions never stops.
- It is possible to use new computational results in order future calculations (self-adaptivity). In this way the system can generate knowledge in autonomous way.

Finite Number of Steps

- Limits $\lim_{n\to\infty} \frac{1}{n} = 0$
- Convergence of infinite series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ ($p = 2 > 1$ series converges).
- If function f is integrable, then the sequence of Riemann sum $\sum_{n=1}^{N} f(x_n)\Delta x_n$ converges to $\int_a^b f(x)dx$ (for appropriate partitions).

Presented methodology can be applied to many scientific theories (mathematics, engineering, chemistry, biology etc.) with finite number of steps.

What is Special About This Research?

All calculations are done in fully **autonomous** way.

Why autonomous calculations?

What is Special About This Research?

All calculations are done in fully **autonomous** way.

Why autonomous calculations?

Correctness

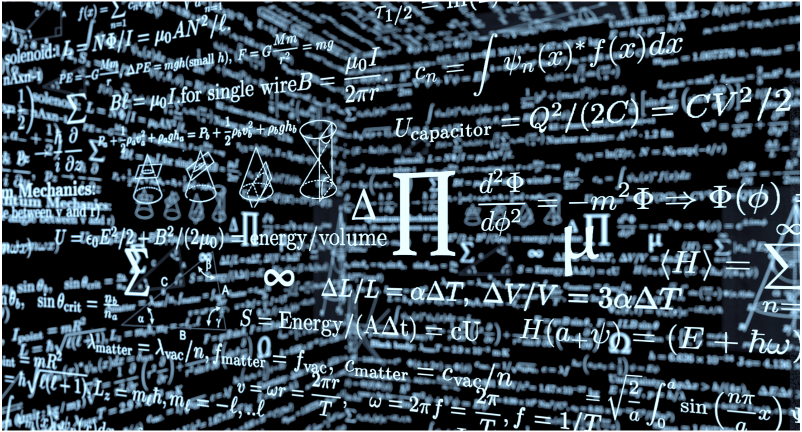
and

Scalability.

Scalability

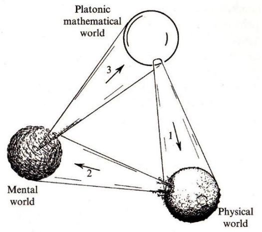
- Moscoso del Pradon uses his method to determine how much information the brain can process during lexical decision tasks. The answer? No more than about 60 bits per second (Emerging Technology, New Measure of Human Brain Processing Speed, MIT Technology Review, August 25, 2009).
- Japanese Fugaku is the world's most powerful supercomputer, reaching 415.53 petaFLOPS (10^{15}) on the LINPACK benchmarks.
- Most PC are in the GigaFLOPS (10^9) range.
- 54-qubit processor, named Sycamore from Google can calculate some tasks in 200 seconds. The worlds fastest supercomputer needs 10000 years to produce a similar output (experiment done in 2019).

Is Mathematics Invented or Discovered?



Mathematical Realism

- Mathematical entities exist **independently of the human mind**.
- Humans do not invent mathematics, but rather **discover it**, and any other intelligent beings in the universe would presumably do the same.



Logicism

- **Logicism** is the thesis that **mathematics is reducible to logic**, and hence nothing but a part of logic.
- Logicists hold that mathematics can be known a priori, but suggest that our knowledge of mathematics is just part of our knowledge of logic in general, and is thus analytic, not requiring any special faculty of mathematical intuition. In this view, logic is the proper foundation of mathematics, and all mathematical statements are necessary logical truths.

Logicism

Rudolf Carnap (1931) presents the logicist thesis in two parts:

- The concepts of mathematics can be derived from logical concepts through explicit definitions.
- The theorems of mathematics can be derived from logical axioms through purely logical deduction.

Hilbert's program.

- Hilbert proposed to ground all existing theories to a finite, complete set of axioms, and provide a proof that these axioms were consistent. Hilbert proposed that the consistency of more complicated systems, such as real analysis, could be proven in terms of simpler systems. Ultimately, the consistency of all of mathematics could be reduced to basic arithmetic.

Logicism

Gödel's incompleteness theorems:

- The first incompleteness theorem states that no consistent system of axioms whose theorems can be listed by an effective procedure (i.e., an algorithm) is capable of proving all truths about the arithmetic of the natural numbers.
- The second incompleteness theorem, an extension of the first, shows that the system cannot demonstrate its own consistency.

Tarski's undefinability theorem: arithmetical truth cannot be defined in arithmetic.

Formalism

- Formalism holds that **mathematical statements may be thought of as statements about the consequences of certain string manipulation rules**.
- According to formalism, mathematical truths are not about numbers and sets and triangles and the likein fact, they are not "about" **anything at all**.
- (Deductivism) Formalism need not mean that mathematics is nothing more than a meaningless symbolic game. It is usually hoped that there exists some interpretation in which the rules of the game hold.

Conventionalism

- Poincaré's use of non-Euclidean geometries in his work on differential equations convinced him that Euclidean geometry should not be regarded as a priori truth. He held that **axioms in geometry should be chosen for the results they produce, not for their apparent coherence with human intuitions about the physical world**.

Mathematical Intuitionism

- In mathematics, intuitionism is a program of methodological reform whose motto is that "**there are no non-experienced mathematical truths**" (L. E. J. Brouwer).
- A major force behind intuitionism was L. E. J. Brouwer, who rejected the usefulness of formalized logic of any sort for mathematics. His student Arend Heyting postulated an intuitionistic logic, different from the classical Aristotelian logic; this logic does not contain the law of the excluded middle and therefore frowns upon proofs by contradiction. The axiom of choice is also rejected in most intuitionistic set theories, though in some versions it is accepted.

Other Theories

Etc.

Mathematics as a Language of Science

I am NOT working on that

Real world object

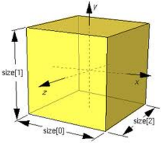
Difficult

abstraction

scientist

I am working on that

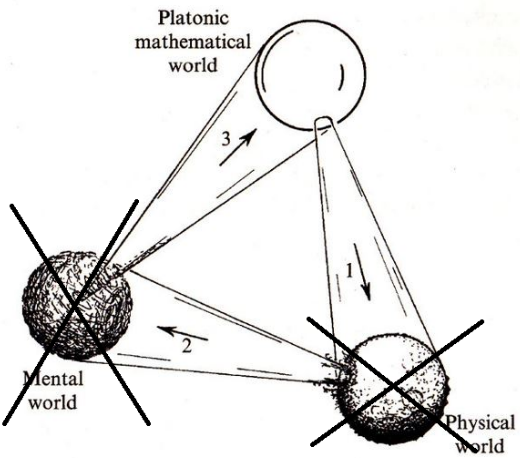
Abstract object



Easy

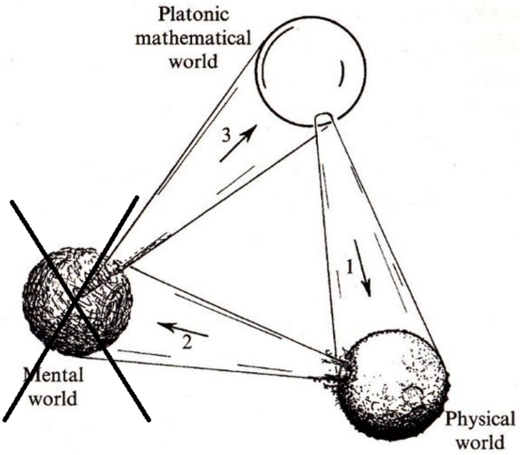
Game Theory
Autonomous Computational Methods
Algebraic Equations
Generalizations and Possible Limitations
Autonomous Computational Methods
Machine Learning
Conclusions

Current version of the software (Plato, 5th century BC).



Autonomous Computational Methods

Future version of the software.

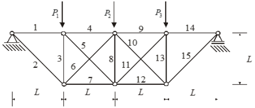


Mathematics as a Language of Science

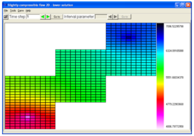
Significant part of engineering problems can be described by using mathematics (and appropriate software).

- Engineering

Mathematics



$$\frac{d}{dx}\left(E_i A_i \frac{du_i}{dx}\right) + q_i = 0$$



$$\frac{\partial}{\partial x}\left[\beta_c k_x A_x \frac{k_{ro}}{\mu_o B_o}\frac{\partial p_o}{\partial x}\right]\Delta x + \frac{\partial}{\partial y}\left[\beta_c k_y A_y \frac{k_{ro}}{\mu_o B_o}\frac{\partial p_o}{\partial y}\right]\Delta y + \frac{\partial}{\partial x}\left[\beta_c k_x A_x \frac{k_{ro}}{\mu_o B_o}\frac{\partial p_o}{\partial x}\right]\Delta z = \frac{V_b}{\alpha_c}\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) - q_{osc}$$

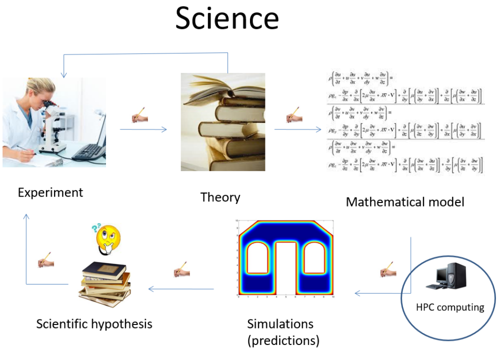
$$\frac{\partial}{\partial x}\left[\beta_c k_x A_x \frac{k_{rw}}{\mu_w B_w}\frac{\partial p_w}{\partial x}\right]\Delta x + \frac{\partial}{\partial y}\left[\beta_c k_y A_y \frac{k_{rw}}{\mu_w B_w}\frac{\partial p_w}{\partial y}\right]\Delta y + \frac{\partial}{\partial x}\left[\beta_c k_x A_x \frac{k_{rw}}{\mu_w B_w}\frac{\partial p_w}{\partial x}\right]\Delta z = \frac{V_b}{\alpha_c}\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) - q_{wsc}$$

$$S_o + S_w = 1$$

$$P_{cow}(S_w) = p_o - p_w$$

Mathematics as a Language of Science

Significant part of engineering problems can be described by using mathematics (and appropriate software).



Solution of Algebraic Equations - - Deterministic Approach

- Linear equation $ax = b$
- Quadratic equation $ax^2 + bx + c = 0$
- Cubic equation $ax^3 + bx^2 + cx + d = 0$
- Non-elementary solutions $x \cdot e^x = 1$
- etc.

Appropriate method of solution can be applied to appropriate equation.

$$Known\ equation \rightarrow Method\ for\ solution$$

Machine Learning in Game Theory

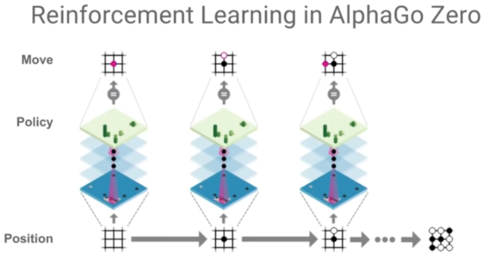


Figure: Machine Learning model of chess (Deepmind AlphaZero)

Problem space

- Trees with up to n internal nodes.
- Set p_1 of unary operators (e.g. cos, sin, exp, log).
- Set p_2 of binary operators (e.g. $+, -, /, \times$).
- a set of L leaf values containing variables (e.g. x, y, z), constants (e.g. e, π), integers (e.g. {,-10,..., 10,}).

The number of binary trees with n internal nodes is given by the n-th Catalan numbers.
A binary tree with n internal nodes has exactly $n+1$ leaves. Each node and leaf can take respectively p_2 and L different values. As a result, the number of expressions with n binary operators can be expressed by:

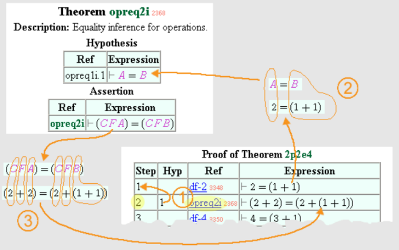
$$E_n = \frac{1}{n+1}\binom{2n}{n} p_n^n L^{n+1} \approx \frac{4^n}{n\sqrt{\pi \cdot n}}(p_2)^n L^{n+1}$$

Solution of Algebraic Equations - - Machine Learning

- It is possible to classify equations by using the neural networks.
- The initial guess provided by the neural networks can be used to find approximate solution of the problem.
- Training of the neural network can be done in fully autonomous way.
- The structure and features of the neural network can be crated in fully autonomous way.
- Initial guess provided by the probabilistic methods have to be verified in the future calculations.

Existing System for Theorem Proving

- Metamath (http://us.metamath.org/)

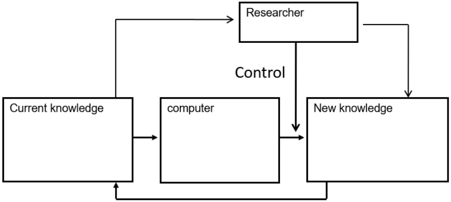


- Mizar system (formal language for writing mathematical definitions and proofs).
- Step by step solutions in Mathematica (WolframAlpha).

Finite Number of Steps? Logic?

- The fundamental difference between if-else and machine learning is that if-else models are static deterministic environments and machine learning (ML) algorithms, are probabilistic stochastic environments.
- Cyc - is the world's longest-lived artificial intelligence project, attempting to assemble a comprehensive ontology and knowledge base. Douglas Lenat began the project in July 1984 at MCC, where he was Principal Scientist 19841994, and then, since January 1995, has been under active development by the Cycorp company, where he is the CEO.

Self-Adaptive Methods



Due to complexity of the problems the process of updating of the knowledge base never stops. The program runs practically indefinitely constantly improving his own computations. Calculations can be speed up by using parallel and distributed computing.

Conclusions

- Mathematical/scientific knowledge can be treated as independent units that can interact with each-other and create new, possibly useful knowledge.
- Generation of new knowledge can be fully automated and autonomous. No interaction with humans is necessary.
- Development of new knowledge is possible in many different fields (e.g. engineering, computer science, mathematics etc.).

Conclusions (continued)

- By using presented methodology it is possible to create complex examples and appropriate computational methods relevant to many areas of mathematics as well as in other areas of science and engineering.
- Machine learning (the main computational method is NOT based on machine learning) can be used as source of good initial guess for processing mathematical information. Actually there is NO main computational method in the system.

Conclusions (continued)

- Once the information is available in the system it will NEVER be forgotten and can be used for generation of new mathematical theorems. From that perspective the system can be viewed as self-organizing archive of information.
- Development of this and similar systems should speed up cooperation among scientists around the world (theoretical possibility).

Conclusions (continued)

- Calculations can be done in distributed way (this option is experimental at this moment). Large number of computers can process the information simultaneously in order to get the results faster. Calculations do not require existence of any centralized system.
- Parallel computing can significantly speed up the calculations (future work).
- Autonomous interaction with external sources of information extends internal database of information and should increase productivity of the system (future work).

Thank You