

Goal oriented self-adaptive development of online assignments based on autonomous computational methods

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Outline

- 1 Online Learning
- 2 Formulation of Scientific Problems
- 3 Mathematical Modeling
- 4 How to Create Online Assignments?
- 5 Tools for Web Development
- 6 Automated Development of Online Assignments
- 7 Goal Oriented Self-adaptive Development of Online Assignments
- 8 Conclusions

Online Learning

Online Learning

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Blackboard

HELP

Enter Blackboard Central's virtual office Mon - Fri 8AM - 5PM
24/7 Blackboard Support 915-747-HELP option # 2
Blackboard Help Website
Provide your Blackboard feedback

Technology Support Helpdesk

MON-THU 7AM-9PM FRI 7AM-8PM SAT 9AM-2PM SUN 12PM-5PM
Visit us at helpdesk.utep.edu
Submit a request
Chat with us
Contact us: 915-747-HELP, helpdesk@utep.edu

TECHNOLOGY Support

Figure: Blackboard

WebAssign

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1. 0/7.69 points

LarCalc11 7.1.006. [3865162]

Write a definite integral that represents the area of the region. (Do not evaluate the integral.)

$$y_1 = x^2 + 2x + 4$$

$$y_2 = 2x + 20$$

$$\int_{-4}^{\boxed{4}} \left(\boxed{4} - (\boxed{16 - x^2}) \right) dx$$

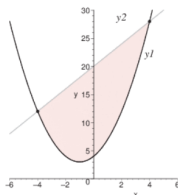


Figure: Sample assignment

Moodle

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Figure: Online learning platform

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The screenshot displays the WeBWorK web interface. At the top, the WeBWorK logo is on the left, and the user is logged in as a student with a 'Log Out' link on the right. Below the header, a navigation bar shows the path: 'WeBWorK → TestCourse → HW3 → 1'. The main content area is titled 'HW3: Problem 1' and indicates a value of '(1 pt) Find'. The problem statement asks for a specific function $F(x)$ such that $F'(x) = \int x^5 \cos(x^6) dx$. Below this, there is a text input field for $F(x) =$. The interface also includes a 'Problems' sidebar with a list of problems (1-7), where Problem 7 is selected. Another sidebar shows 'Display Options' with radio buttons for 'plainText', 'formattedText', 'images', 'jsMath', 'asciimath', and 'LaTeXMathML'. At the bottom, there are buttons for 'Preview Answers', 'Submit Answers', and 'Email instructor'. A footer note states: 'Page generated at 10:35am on May 12, 2008. WeBWorK © 2000-2006 The WeBWorK Project'.

WebWork

Logged in as student.
[Log Out](#)

WebWork → TestCourse → HW3 → 1

▲ Prob. List Next ►

HW3: Problem 1

(1 pt) Find

$$F(x) = \int x^5 \cos(x^6) dx$$

Give a specific function for $F(x)$

$F(x) =$

Evaluate

$$F(x) = \int_{-1}^4 x^5 \cos(x^6) dx$$

[Preview Answers](#) [Submit Answers](#)

You have attempted this problem 0 times.
You have unlimited attempts remaining.

[Email instructor](#)

Page generated at 10:35am on May 12, 2008
WeBWorK © 2000-2006 [The WeBWorK Project](#)

Figure: Sample assignment

Early Examples of Distance Learning

Online Learning

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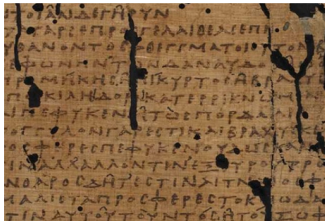
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User Main Page

	Logoff
Username:	80080081
First Name:	Pownuk
Last Name:	Andrew
Group:	2020-Fall-MATH-1312-CRN-12219
	Change password
	Change e-mail

Show my grades

Files (notes, syllabus etc.)

List of Students

Online Learning

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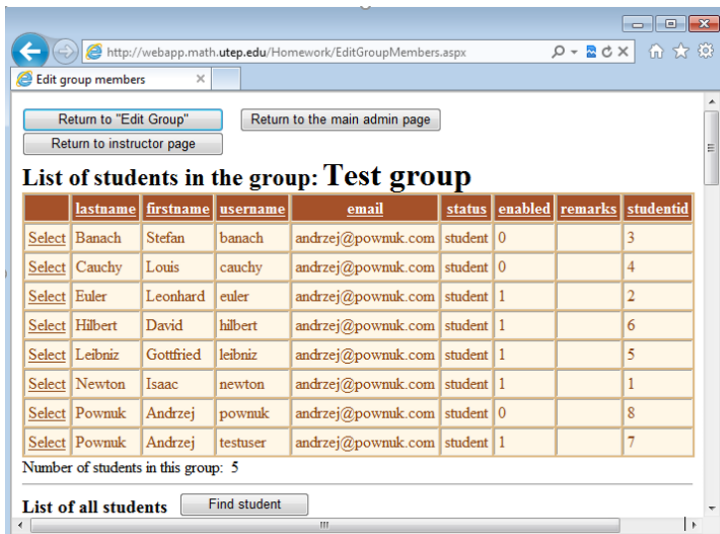
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The screenshot shows a web browser window with the address bar displaying `http://webapp.math.utep.edu/Homework/EditGroupMembers.aspx`. The page title is "Edit group members". At the top, there are three buttons: "Return to 'Edit Group'", "Return to the main admin page", and "Return to instructor page". Below these buttons, the heading "List of students in the group: Test group" is displayed. A table follows, listing students with columns for selection, last name, first name, username, email, status, enabled status, remarks, and student ID. The table contains 8 rows of data. Below the table, it states "Number of students in this group: 5". At the bottom, there is a section titled "List of all students" with a "Find student" button.

	lastname	firstname	username	email	status	enabled	remarks	studentid
Select	Banach	Stefan	banach	andrzej@pownuk.com	student	0		3
Select	Cauchy	Louis	cauchy	andrzej@pownuk.com	student	0		4
Select	Euler	Leonhard	euler	andrzej@pownuk.com	student	1		2
Select	Hilbert	David	hilbert	andrzej@pownuk.com	student	1		6
Select	Leibniz	Gottfried	leibniz	andrzej@pownuk.com	student	1		5
Select	Newton	Isaac	newton	andrzej@pownuk.com	student	1		1
Select	Pownuk	Andrzej	pownuk	andrzej@pownuk.com	student	0		8
Select	Pownuk	Andrzej	testuser	andrzej@pownuk.com	student	1		7

Number of students in this group: 5

List of all students

Information About Particular Student

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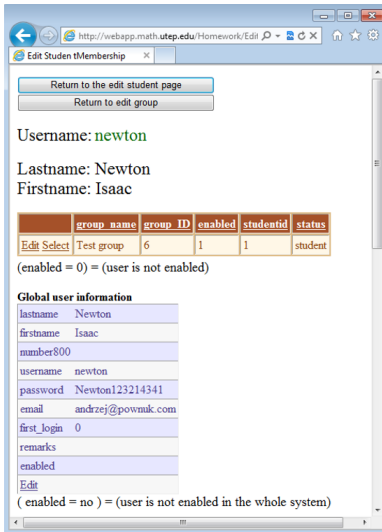
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The screenshot shows a web browser window with the address bar displaying `http://webapp.math.utep.edu/Homework/Edit`. The page title is "Edit Student Membership". At the top, there are two buttons: "Return to the edit student page" and "Return to edit group". Below these, the student's information is displayed: Username: newton, Lastname: Newton, and Firstname: Isaac. A table follows, showing a list of groups. The table has columns: group_name, group_ID, enabled, studentid, and status. The first row shows "Test group" with group_ID 6, enabled status 1, studentid 1, and status "student". Below the table, a note states "(enabled = 0) = (user is not enabled)". Further down, a section titled "Global user information" contains a table with fields: lastname (Newton), firstname (Isaac), number800, username (newton), password (Newton123214341), email (andrzej@pownuk.com), first_login (0), remarks, and enabled. At the bottom of this section is an "Edit" link. A final note at the bottom of the page states "(enabled = no) = (user is not enabled in the whole system)".

Return to the edit student page

Return to edit group

Username: newton

Lastname: Newton

Firstname: Isaac

	group_name	group_ID	enabled	studentid	status
Edit Select	Test group	6	1	1	student

(enabled = 0) = (user is not enabled)

Global user information

lastname	Newton
firstname	Isaac
number800	
username	newton
password	Newton123214341
email	andrzej@pownuk.com
first_login	0
remarks	
enabled	

[Edit](#)

(enabled = no) = (user is not enabled in the whole system)

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http://webapp.math.utep.edu/Homework/EditHomeworkList.aspx

Untitled Page

Edit Select	ID	Homework Title	File Path
Edit Select	42	TG-Homework-2	/HomeworkDir/TG-Homework-2.aspx
Edit Select	43	Cal-III-Homework-13	/HomeworkDir/Cal-III-Homework-13.aspx
Edit Select	44	Cal-II-Homework-12	/HomeworkDir/Cal-II-Homework-12.aspx
Edit Select	45	AA-I-Homework-12	/HomeworkDir/AA-I-Homework-12.aspx
Edit Select	46	Cal-III-Homework-14	/HomeworkDir/Cal-III-Homework-14.aspx
Edit Select	47	Cal-II-Homework-13	/HomeworkDir/Cal-II-Homework-13.aspx
Edit Select	48	AA-I-Homework-13	/HomeworkDir/AA-I-Homework-13.aspx
Edit Select	49	AA-I-Homework-14	/HomeworkDir/AA-I-Homework-14.aspx
Edit Select	50	AA-I-Homework-15	/HomeworkDir/AA-I-Homework-15.aspx

1 2 3 4

Find the Laurent series for

$$f(z) = \frac{\sin(2z)}{z^2}$$

at $z_0 = 0$

$a_{-1} =$

$a_0 =$

Calculate and submit grade

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Untitled Page - Windows Internet Explorer

http://localhost:54348/Homework/HomeworkDir/Cal-III-Homework-4.aspx

Live Search

Untitled Page

Calculate the equation of plane that passess through the following points

A=(1,0,0)
B=(0,1,0)
C=(0,0,-2)

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = [\text{ } , \text{ } , \text{ }]$

Equation of plain (for example $x+y-z-2=0$)

=0

(symbolic formula)

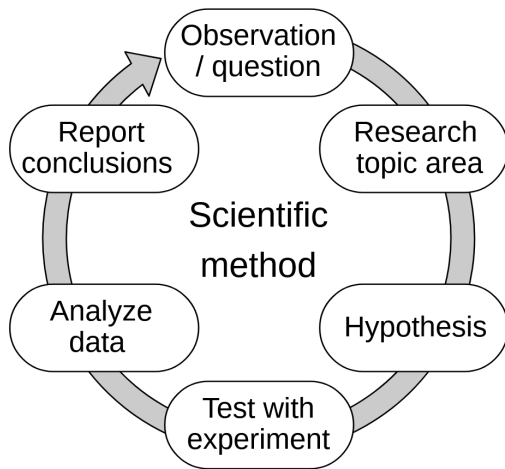
Submit the answer

Done

Local intranet | Protected Mode: On

100%

Scientific Method (Aristotle 384–322 BCE)



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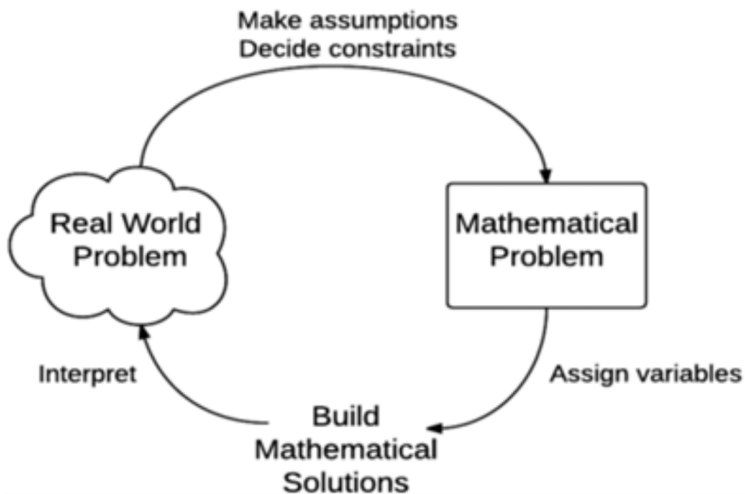
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Curry–Howard Correspondence

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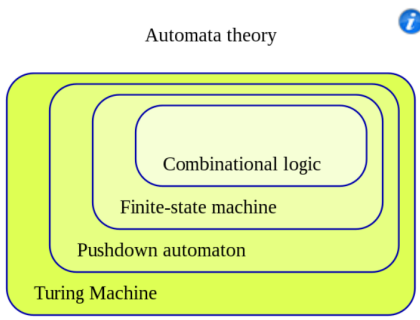
Conclusions

The Curry–Howard correspondence (also known as the Curry–Howard isomorphism or equivalence, or the proofs-as-programs and propositions- or formulate-as-types interpretation) is the direct relationship between computer programs and mathematical proofs.

Teacher/scientists can use this relation and create computer programs from mathematical formulations and mathematical formulations from computer programs.

Turing Machine

A Turing machine is a mathematical model of computation describing an abstract machine that manipulates symbols on a strip of tape according to a table of rules. Despite the model's simplicity, it is capable of implementing any computer algorithm.



Turing Complete Programming Languages in Teaching

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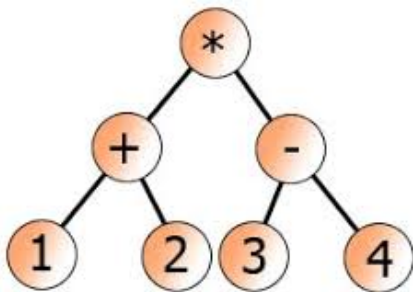
Conclusions

Any Turing complete programming language (e.g. C,C++,Python, Java etc.) can express wide class of algorithms.

This gives teachers a tool to work interactively online on problems with practically arbitrary complexity.

To use this tool, it is necessary to represent a mathematical problem in a form of computer code and then the students by using webpages can interact with this code to check their progress. In the same way it is possible to work with research problems with arbitrary complexity.

Expression Tree



$((1+2)*(3-4))$

How to Evaluate Mathematical Expression Given as a String?

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Example

- Expression: $\text{sqrt}(2)*6$
- Value: $\sqrt{2} \cdot 6$

Example

- Expression: $\text{sqrt}2*6$
- Value: $\sqrt{2} \cdot 6$

Example

- Expression: $\text{Sqrt}[2]6$
- Value: $\sqrt{2} \cdot 6$

Example

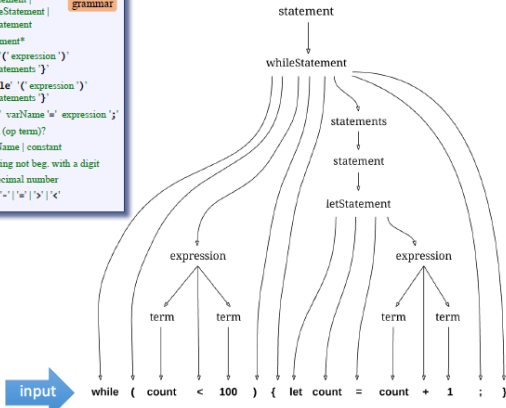
- Expression: $\text{SQRT}[2]6$
- Value: $\sqrt{2} \cdot 6$

Parse Tree

Parse tree

grammar

```
statement: ifStatement |
          whileStatement |
          letStatement
statements: statement*
ifStatement: 'if' '(' expression ')'
            '(' statements ')'
whileStatement: 'while' '(' expression ')'
              '(' statements ')'
letStatement: 'let' varName 'u' expression ';'
expression: term (op term)?
term: varName | constant
varName: a string not beg. with a digit
constant: a decimal number
op: '+' | '-' | '*' | '/' | '<'
```



XML Parse Tree

Parse tree

```
statement: ifStatement |  
          whileStatement |  
          letStatement  
statement: statement*  
ifStatement: 'if' '(' expression ')' '  
' statements ')'  
whileStatement: 'while' '(' expression ')' '  
' statements ')'  
letStatement: 'let' varName '=' expression ';' '  
expression: term (op term)?  
term: varName | constant  
varName: a string not beg. with a digit  
constant: a decimal number  
op: '+' '-' '*' '/' '>' '<'
```

grammar

```
<whileStatement>  
<keyword> while </keyword>  
<symbol> ( </symbol>  
<expression>  
  <term>  
    <identifier> count </identifier>  
  </term>  
  <symbol> < </symbol>  
  <term>  
    <IntConstant> 100 </IntConstant>  
  </term>  
</expression>  
<symbol> ) </symbol>  
<symbol> { </symbol>  
<statements>  
  <letStatement>  
    <keyword> let </keyword>  
    <identifier> count </identifier>  
    <symbol> = </symbol>  
    <expression>  
      <term> <identifier> count </identifier> </term>  
      <symbol> + </symbol>  
      <term> <IntConstant> 1 </IntConstant> </term>  
    </expression>  
    <symbol> ; </symbol>  
  </letStatement>  
</statements>  
<symbol> } </symbol>  
</whileStatement>
```

parser output

Same parse tree,
in XML

Grammar

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Lexical elements:	The Jack language includes five types of terminal elements (tokens):
keyword:	'class' 'constructor' 'function' 'method' 'field' 'static' 'var' 'int' 'char' 'boolean' 'void' 'true' 'false' 'null' 'this' 'let' 'do' 'if' 'else' 'while' 'return'
symbol:	'(' ')' '[' ']' '{' '}' '.' ',' ';' ':' '=' '%' '*' '/' '<' '>' '<=' '>=' '~'
integerConstant:	A decimal number in the range 0 .. 32767.
StringConstant:	"" A sequence of Unicode characters not including double quote or newline ""
identifier:	A sequence of letters, digits, and underscore ('_') not starting with a digit.
Program structure:	A Jack program is a collection of classes, each appearing in a separate file. The compilation unit is a class. A class is a sequence of tokens structured according to the following context free syntax:
class:	'class' className '(' 'classVarDec*' subroutineDec* ')'
classVarDec:	('static' 'field') type varName (',' varName)* ';'
type:	'int' 'char' 'boolean' className
subroutineDec:	('constructor' 'function' 'method') ('void' type) subroutineName '(' parameterList ')' subroutineBody
parameterList:	('(' type varName ')' ',' type varName)* ')'
subroutineBody:	'{' varDec* statements '}'
varDec:	'var' type varName (',' varName)* ';'
className:	identifier
subroutineName:	identifier
varName:	identifier
Statements:	
statements:	statement*
statement:	letStatement ifStatement whileStatement doStatement returnStatement
letStatement:	'let' varName '(' 'I' expression ')' '=' expression ';'
ifStatement:	'if' '(' expression ')' 'I' statements 'I' ('else' 'I' statements ')' '}'
whileStatement:	'while' '(' expression ')' 'I' statements 'I' }
doStatement:	'do' subroutineCall ';'
ReturnStatement:	'return' expression? ';'
Expressions:	
expression:	term (op term)*
term:	integerConstant stringConstant keywordConstant varName varName '(' 'I' expression ')' subroutineCall '(' 'I' expression ')' unaryOp term
subroutineCall:	subroutineName '(' (expressionList ')' (className varName) ',' subroutineName '(' expressionList ')')
expressionList:	(expression ',' expression)* '}'
op:	'+' '-' '*' '/' '%' '<' '>' '<=' '>=' '~'
unaryOp:	'~' '~'
KeywordConstant:	'true' 'false' 'null' 'this'

Infix Notation, Prefix Notation, Postfix Notation

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Different notation for arithmetic expressions.

- Infix notation $(5 + 6) \times 7$
- Prefix notation $x + 567$
- Postfix notation $756 + x$

Typical evaluation process of arithmetic expressions.

- $\text{InfixToPrefix}((5 + 6) \times 7) = x + 567$
- $\text{EvaluatePrefix}(x + 567) = 77$

WebAssign Question Modes

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Mode	Description
Algebraic	Students submit a mathematical expression or equation that is evaluated algebraically.
Essay	Students submit an extended textual response. Scored manually.
File-Upload	Students submit a file. Scored manually.
Fill-in-the-Blank	Students submit a brief textual response.
Graphing	Students draw on a Cartesian coordinate plane.
Image Map	Students click a displayed image.
Java	Students use Java, Flash, or other applet to answer the question.
Matching	Students match items from two lists.
Multiple-Choice	Students select one response from a list.
Multiple-Select	Students select one or more responses from a list.
NumberLine	Students place or draw points, lines, segments, or rays.
Numerical	Students submit a numerical response which might include units or be checked for significant figures.
Poll	Used with other modes to create questions that collect information only. All responses are scored correct.
Symbolic	Students submit a mathematical expression (not an equation) that is evaluated by value substitution.

WebAssign Questions from Code (HTML, Perl etc.)

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The following code in Answer accepts two values between 20 and 100, and then requires your students to multiply them:

```
<EQN $A=userinput(20,100,50)>  
<EQN $B=userinput(20,100,50)>  
<EQN $SIMPLIFIED=1; $A * $B>
```

To require students to perform the computations instead of stating their response as a mathematical expression, set the \$SIMPLIFIED, \$FRACTION, or \$PROPERFRACTION variables to 1 as you would for any other numerical question.

TCP/IP

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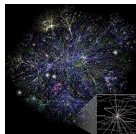
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The Internet protocol suite, commonly known as TCP/IP, is a framework of organizing the set of communication protocols used in the Internet and similar computer networks according to functional criteria. The foundational protocols in the suite are the Transmission Control Protocol (TCP), the User Datagram Protocol (UDP), and the Internet Protocol (IP). In the development of this networking model, early versions of it were known as the Department of Defense (DoD) model because the research and development were funded by the United States Department of Defense through DARPA. ARPANET started in 1969.



HTML

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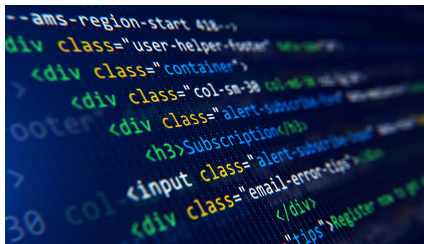
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The HyperText Markup Language or HTML is the standard markup language for documents designed to be displayed in a web browser. It can be assisted by technologies such as Cascading Style Sheets (CSS) and scripting languages such as JavaScript. Initial release 1993, Tim Berners-Lee, CERN (European Organization for Nuclear Research).



PHP

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Paradigm	Multi-paradigm: imperative, functional, object-oriented, procedural, reflective
Designed by	Rasmus Lerdorf
Developer	The PHP Development Team, Zend Technologies , PHP Foundation ↗
First appeared	June 8, 1995; 27 years ago ^{[1][2]}
Stable release	8.1.12 / 2022-10-25
Preview release	8.2.0 beta / 2022-07-21
Typing discipline	Dynamic, weak, gradual ^[3]
Implementation language	C (primarily; some components C++)
OS	Unix-like, Windows, macOS, IBM i, OpenVMS
License	PHP License (most of Zend engine

JavaScript

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Paradigm	Multi-paradigm: event-driven, functional, imperative, procedural, object-oriented programming
Designed by	Brendan Eich of Netscape initially; others have also contributed to the ECMAScript standard
First appeared	December 4, 1995; 26 years ago ^[1]
Stable release	ECMAScript 2021 ^[2]  / June 2021; 16 months ago
Preview release	ECMAScript 2022 ^[3]  / 22 July 2021; 15 months ago
Typing discipline	Dynamic, weak, duck
Filename extensions	<code>.js</code> · <code>.cjs</code> · <code>.mjs</code> ^[4]

Asp.Net

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Developer(s)	Microsoft
Initial release	January 5, 2002; 20 years ago
Stable release	6 / February 8, 2022; 8 months ago ^[1]
Written in	.NET languages
Operating system	Microsoft Windows, Linux, macOS
Platform	Cross-platform
Type	Web framework
License	Apache License 2.0
Website	www.asp.net 

Automated Development of Mathematical Description

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- General formulation of the problem.
- Formulation of possible special cases.
- Construction of the search space for a given problem.
- Finding possible limitations and stopping criteria.

$$\frac{d}{dx} \cos(x) + \frac{d}{dx} (2 + \cos(\cos(x))) \quad (708)$$

$$\frac{d}{dx} \cos(x) + \frac{d}{dx} (2 + (2 + 2)) \quad (709)$$

$$\frac{d}{dx} \cos(x) + \frac{d}{dx} (2 + (2 + x)) \quad (710)$$

$$\frac{d}{dx} \cos(x) + \frac{d}{dx} (2 + (2 + \sin(x))) \quad (711)$$

$$\frac{d}{dx} \cos(x) + \frac{d}{dx} (2 + (2 + \cos(x))) \quad (712)$$

$$\frac{d}{dx} \cos(x) + \frac{d}{dx} (2 + (x + x)) \quad (713)$$

Automated Development of Step-by-step Solution

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Conclusions

step-1

$$((1 + 2) + (2 + 1)) = x$$

step-2

$$((1 + 2) + (2 + 1)) = x$$

step-3

$$(3 + (2 + 1)) = x$$

step-4

$$((2 + 1) + 1 + 2) = x$$

step-5

$$((1 + 2) + 3) = x$$

step-6

$$((1 + 2) + 2 + 1) = x$$

step-7

$$(3 + (2 + 1)) = x$$

step-8

$$((2 + 1) + 1 + 2) = x$$

step-9

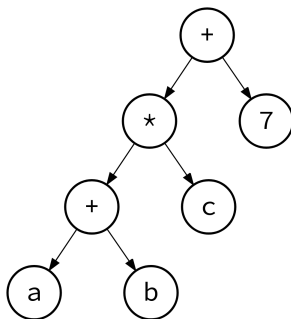
$$((1 + 2) + 3) = x$$

step-10

$$((1 + 2) + 2 + 1) = x$$

Conversion to Computer Code

- Mathematical formulation can be represented by a computational graphs.
- Graphs can be translated into computer code in given programming language.



Uploading Code to the Server

```
#!/bin/bash
HOST="hostname"
USER="username"
PASSWORD="password"
DESTINATION=$1
NEWDIR=$2
FILE=$3
ftp -inv $HOST <<EOF
user $USER $PASSWORD
cd $DESTINATION
pwd
mkdir $NEWDIR
cd $NEWDIR
pwd
passive
mput *.aspx
```

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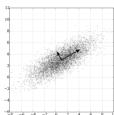
Conclusions

- In some cases it is possible to construct very specific computational graph which is related to given problem.
- Computational graph for a given problem can be represented in many different ways.
- Family of computational graphs can be translated into specific computer code.
- Computer code can be embedded into existing online learning framework and formulated as an assignment for student.

Example - Finding Eigenvalues

- Generate a matrix A with known eigenvalues.
- Write characteristic equation.
$$a_n \lambda^n + a_{n-1} \lambda^{n-1} + \dots + a_1 \lambda + a_0 = 0$$
- Find eigenvalues $\lambda_1, \dots, \lambda_n$.
- Find related eigenvectors and generalized eigenvectors (v_1, v_2, \dots, v_n) and construct related Jordan form of the matrix $J = P^{-1}AP$.
- Apply the primary decomposition theorem
$$S = N((A - \lambda_1 I)^{k_1}) \oplus \dots \oplus N((A - \lambda_m I)^{k_m})$$

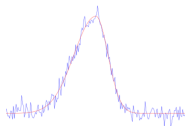
Presented problem has many special cases and related assignments require appropriate software which controls steps of the calculations.



Example - Discrete Least Square Approximation

- Specify the model $\phi(x, w)$ ($w = (w_1, w_2, \dots, w_m)$).
- Specify the loss function $L(x, y, \phi(x, w))$ for given discrete set of values $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n)$.
- Specify an algorithm for finding $w = \arg \min_w L(x, y, \phi(x, w))$
- Find parameters w and specify the model $\phi(x, w)$.

In presented assignment it is possible to consider various loss functions and models ϕ (polynomial approximation, neural networks, etc.) and optimization algorithms (direct optimization, gradient decent, stochastic gradient descent etc.).



Example - Weak Formulation and FEM Method

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Conclusions

For the following boundary value problem

$$\frac{d}{dx} \left(EA \frac{du}{dx} \right) + n = 0, u(0) = 0, u(L) = 0$$

Find related variational equation

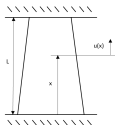
$$a(u, v) = l(v) \forall v \in H_0^1(\Omega)$$

After calculations we know that

$$a(u, v) = \int_0^L EA \frac{du}{dx} \frac{dv}{dx} dx, l(v) = \int_0^L n v dx$$

$$\|u\|_{W^{k,p}(\Omega)} = \left(\sum_{|\alpha| \leq k} \|D^\alpha u\|_{L^p(\Omega)}^p \right)^{\frac{1}{p}}$$

In presented assignment it is possible to consider various approximation spaces $V_h \subset H_0^1(\Omega)$, geometry of the problem, and loads. The Lax-Milgram theorem can be applied to prove existence and uniqueness of the solution.



Numerical Symulations in Linear/Nonlinear Elasticity

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Equation of motion

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{F} = \rho \ddot{\mathbf{u}}$$

Constitutive equations

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{\varepsilon}$ is the infinitesimal strain tensor, \mathbf{u} is the displacement (vector), \mathbf{C} is the fourth-order stiffness tensor, \mathbf{F} is the body force per unit volume, ρ is the mass density.

$$C_{ijkl} = K \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl})$$

$$\sigma_{ij} = K \delta_{ij} \varepsilon_{kk} + 2\mu (\varepsilon_{ij} - \frac{1}{3} \delta_{ij} \varepsilon_{kk})$$

$$\varepsilon_{rs} = \frac{1}{2} \left(\frac{\partial u_r}{\partial x_s} + \frac{\partial u_s}{\partial x_r} + \frac{\partial u_k}{\partial x_r} \frac{\partial u_k}{\partial x_s} \right) \approx \frac{1}{2} \left(\frac{\partial u_r}{\partial x_s} + \frac{\partial u_s}{\partial x_r} \right)$$

In presented assignment it is possible to consider different geometry of the problems, loads, shape functions, approximation spaces etc.

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Conclusions

- Turing complete programming languages give teachers a tool to work interactively online on problems with practically arbitrary complexity.
- By using Turing complete computer languages it is possible to implement very wide class of mathematical problems by using computer code.
- Many engineering problems can be describe by using mathematical models, because of that it is possible to generate automatically online assignments for students.
- It is continent to create a set of goal-oriented assignments.
- In some cases every step of generation of online learning assignments can be fully automated and speed up by using autonomous computational algorithms.

Thank You