Calculating Risk of Cost Using Monte Carlo Simulations with Fuzzy Parameters in Civil Engineering

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Abstract. Risk is a part of almost all civil engineering projects. Usually there is a difference between the real and the estimated cost of the civil engineering projects. Unfortunately, in civil engineering applications usually we do not have enough data to calculate probabilistic characteristics [13]. There are also different methods of modeling of uncertainty [6, 4]. In this paper probabilistic characteristics are modeled by fuzzy numbers, which are defined by some expert. The resulting cost is described by probability density functions with fuzzy characteristics (for example mean or standard deviation). Using assessment from different (or even one) experts we can estimate the uncertainty of the probability density function of total costs and the risk. Then using modified Monte-Carlo simulation and the alpha cut method we can calculate the results.

Keywords: risk, costs, uncertainty, impresise probability

1. Introduction

Risk us an integral part of each civil engineering project. We can define it as possibility of occurrence of loss. One of the most popular type contracts in Poland is (guaranteed maximum price or cost contract) . At this time task and costs are predicted on the basis on deterministic unit costs [13].

Tasks and unit costs are deterministic. Unfortunately, in reality schedule tasks and unit costs may change because of the influence of different and usually uncertain factors[2].

2. Calculating of cost of civil engineering projects

Today in Poland the cost of civil engineering project is calculated by using pure deterministic methods which are based on some catalogues [11], set of prices [8, 9] and/or different norms. Existing practical methods of calculating costs are pure deterministic.

The total cost can be calculated as:

$$c_T = DC + IC + P + T \tag{1}$$

where DC - direct costs (labor, material, equipment), IC - indirect cost (costs of management, cost of insurance etc.), P - profit and risk of the project, T – taxes.

In this paper only direct costs DC are taken into account.

$$DC = \sum_{i=1}^{n} DC_i \tag{2}$$

where DC_i direct costs of each tasks.

3. Probabilistic definition of risk

Many variables have impact upon cost overruns. The prime variables have been commonly identified as: unpredictable weather, inflationary material cost, inaccurate materials estimates, complexity of project, contractor's lack experience, poor labor productivity, project changes [10, 6].

The risk of cost is equal to the probability that the real cost c_T is grater than assumed cost $c_{T,0}$ (maximal).

$$R = P\left\{c_T > c_{T,0}\right\} = 1 - P\left\{c_T \leq c_{T,0}\right\} \tag{3}$$

If we know the probability density function $f_{c_T}(x)$ of the random variable c_T then

$$R = 1 - \int_{-\infty}^{c_{T,0}} f_{c_T}(x)\, dx = 1 - \Phi_{c_T}(c_{T,0}) \tag{4}$$

where

$$\Phi_{c_T}(x) = \int_{-\infty}^{x} f_{c_T}(t)\, dt \tag{5}$$

is a cumulative distribution function of the random variable c_T.

It should be emphasize that the influence of the uncertainty to the final cost is very difficult to estimate by using pure probabilistic methods due to lack of credible statistical data.

4. Calculating of risk of direct costs

At this moment the direst costs are calculated on the basis on standards [13] which are very general and they do not take into account different factors which have influence on their values. Because of that there is a difference between the real costs and predicted costs.

Risk is calculated as constant value which is introduced in order to cover the losses. The final result of the calculation is a fixed value.

The final price is a result of negotiation between investor and contractors.

Important information for the contractor is the following:

- what is the level of risk which accompany assumed maximal level of direct costs.

- what is the minimal cost for which the risk can be accepted.

Knowledge about influence of random parameters of the system would be a very good in negotiations.

There are many programs which enable to calculate project risk (for example Pert Master, Risk, MS Project etc.) in pure probabilistic sense. However in practice it is very difficult to obtain reliable statistical data, because of that the results of the calculations are not credible.

5. Example of analysis of risk

Let's assume that contractor would like to realize some civil engineering project for fixed price. The project consist of: determine tasks, alternative tasks and additional tasks.

One can called the task deterministic if occurrence of it is certain.

Let's assume that we have two tasks. If in each realization of that process we can get only one of them, then we can call these tasks alternative.

If the task may occur in each realization with some probability then we can call that task additional.

5.1. Preparation of data

Valuation of identified tasks was made on the basis of [8, 9, 11]. Then the data was aggregated with taking into account technology of realization and allocation of risk. The results are presented in the table 1.

The model of the system consists of some node. Each node is characterized by some costc_i. One can also define some relations between the elements. Both route thru the graph and the costs c_i are random. The process can be shown as a Petri nets[14] on the Fig. 1.

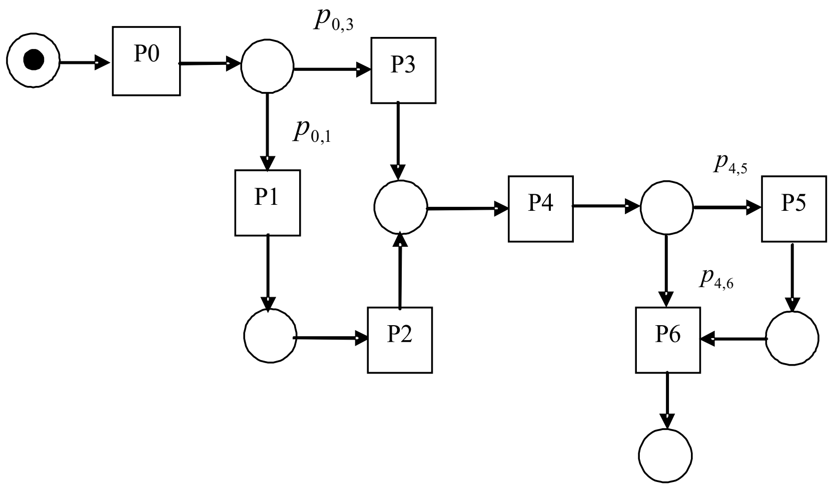

Figure 1. Graphical representation of the process

Table I. Tasks description

No.	Name of tasks	Costs [PLN]	Remarks:
1	**P0**	217.500	deterministic task
2	**P1**	132.000	alternative task with P3
3	**P2**	187.700	alternative task with P3
4	**P3**	420.000	alternative task with P1, P2
5	**P4**	261.700	deterministic task
6	**P5**	43.200	additional task
7	**P6**	125.300	deterministic task

The tasks are represented by rectangles, conditions are represented as circle and the arrows show the direction of movement in the graph. On some connections there is information about the probability of occurrence of each variant.

According to many numerical experiments adequacy of the cost estimation can be characterized by using beta Pert distribution. Beta Pert distribution can be define by using most optimistic cost c_o, most likely cost $c_m$and most pessimistic c_p [3, 12, 1] .

$$\alpha = \frac{4 \cdot (c_m - c_o)}{c_p - c_o}, \quad \beta = 4 - \alpha \tag{6}$$

where α, β are parameters of beta distribution.

$$f_{\alpha,\beta}(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-x)^{\beta-1}x^{\alpha-1}, \quad x \in [0,\ 1] \tag{7}$$

In calculation it is necessary to use the PDF which is defined on the interval $[c_o, c_p]$ i.e.

$$f(x) = \frac{1}{c_p - c_o} f_{\alpha,\beta}\left(\frac{x - c_o}{c_p - c_o}\right). \tag{8}$$

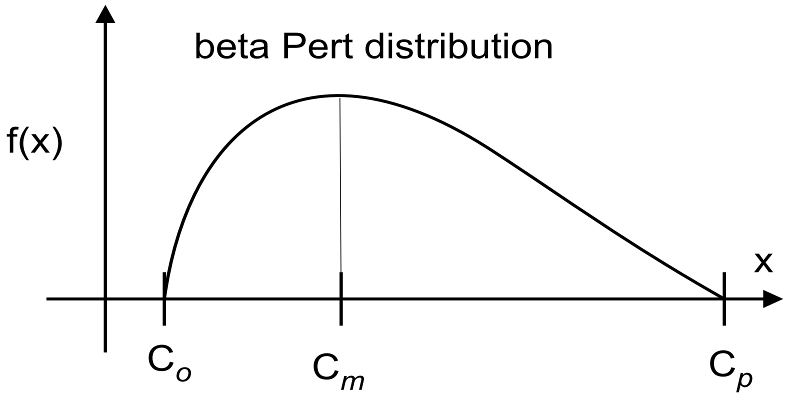

Figure 2. Beta distribution

Beta Pert distribution is widely used to modeling of uncertainty of cost because of it is very intuitive (can be defined using c_o, c_m, c_p).

However usually we do not know the numbers c_o, c_m, c_p precisely. However, usually it is possible to estimate upper and lower bounds its values by using expert knowledge.

$$c_o^- \leq c_o \leq c_o^+, \quad c_m^- \leq c_m \leq c_m^+, \quad c_p^- \leq c_p \leq c_p^+ \tag{9}$$

This information is very imprecise. In order to make the calculations more precisely fuzzy numbers can be applied.

Let's assume that we would like to define fuzzy numbers $c_{o,F}, c_{m,F}, c_{p,F}$ which represent the number c_o, c_m, c_p. We assume that we know the expert(syrveyor-E1, planner-E2 , site agent-E3) opinions $c_o(\omega_i), c_m(\omega_i), c_p(\omega_i)$ for each expert $\omega_1, \omega_2, ..., \omega_n \in \Omega$. We can treat ω_i as elementary event of some probability spaceΩ. Examples of such expert opinions are shown in the table 2 and 3. The expert opinions are interval valued $c_p(\omega_i), c_m(\omega_i), c_o(\omega_i)$ or set they are simply numbers i.e.

$$c_p(\omega_i), c_m(\omega_i), c_o(\omega_i) \in I(R) \tag{10}$$

Alpha cut of fuzzy numbers $c_{p,F}, c_{m,F}, c_{o,F}$ can be constructed by using confidence intervals [7]. For given α level the appropriate α cut $c_{p,F,\alpha}, c_{m,F,\alpha}, c_{o,F,\alpha}$ should satisfy the following condition.

$$P\{\omega_i : c_p(\omega_i) \cap c_{p,F,\alpha} \neq \emptyset\} = 1 - \alpha \tag{11}$$

$$P\{\omega_i : c_m(\omega_i) \cap c_{m,F,\alpha} \neq \emptyset\} = 1 - \alpha \tag{12}$$

$$P\{\omega_i : c_o(\omega_i) \cap c_{o,F,\alpha} \neq \emptyset\} = 1 - \alpha \tag{13}$$

In the simplest case it is possible to apply triangular fuzzy numbers which are defined in the following way:

$$c^-_{p,F,0} = min\{c^-_p(\omega_i) : \omega_i \in \Omega\} \tag{14}$$

$$c^+_{p,F,0} = min\{c^+_p(\omega_i) : \omega_i \in \Omega\} \tag{15}$$

$$c^-_{m,F,0} = min\{c^-_m(\omega_i) : \omega_i \in \Omega\}, \tag{16}$$

$$c^+_{m,F,0} = min\{c^+_m(\omega_i) : \omega_i \in \Omega\} \tag{17}$$

$$c^-_{o,F,0} = min\{c^-_o(\omega_i) : \omega_i \in \Omega\}, \tag{18}$$

$$c^+_{o,F,0} = min\{c^+_o(\omega_i) : \omega_i \in \Omega\} \tag{19}$$

The vertex of triangle fuzzy number can be defined using generalized main value.

$$c_{p,F,1} = \sum_{\omega_i \in \Omega} P\{\omega_i\} \cdot mid(c_p(\omega_i)), \tag{20}$$

$$c_{m,F,1} = \sum_{\omega_i \in \Omega} P\{\omega_i\} \cdot mid(c_p(\omega_i)), \tag{21}$$

$$c_{o,F,1} = \sum_{\omega_i \in \Omega} P\{\omega_i\} \cdot mid(c_p(\omega_i)). \tag{22}$$

The fuzzy numbers are which will be used in calculations are given in the table 5 and 6.

The total cost can be calculated as a sum of random variable with uncertain parameters $\mathbf{h} \in \hat{\mathbf{h}}_\alpha$.

$$c_T(\omega, \mathbf{h}) = \sum_{i=1}^{n} \chi_i(\omega, \mathbf{h}) \cdot c_i(\omega, \mathbf{h}) \tag{23}$$

Table II. Table of cost evaluation

Process name: P1			
Cost: 217.500			
Person: E2			
Percent of cost [%]	Min	Mid	Max
75			
80			
85			
90			
95	X		
100			
105		X	
110			
115			X
120			X
125			
130			
135			
140			

where $\chi : \Omega \times \hat{\mathbf{h}}_\alpha (\omega, \mathbf{h}) \to \chi (\omega, \mathbf{h}) \in \{0,\ 1\}$, $c_i : \Omega \times \hat{\mathbf{h}}_\alpha \ni (\omega, \mathbf{h}) \to c_i (\omega, \mathbf{h}) \in R$, $c_T : \Omega \times \hat{\mathbf{h}}_\alpha \ni (\omega, \mathbf{h}) \to c_T (\omega, \mathbf{h}) \in R$ are some random variables with uncertain parameters, $\hat{\mathbf{h}}_\alpha = \left[h_1^-, h_1^+\right] \times \left[h_2^-, h_2^+\right] \times ... \times [h_m^-, h_m^+] \subseteq I (R^m)$ is an interval vector.

Extreme values of the risk can be defined in the following way:

$$\hat{R}_\alpha (c_{T,0}) = \left[R_\alpha^- (c_{T,0}), R_\alpha^+ (c_{T,0})\right] \tag{24}$$

$$\hat{R}_\alpha (c_{T,0}) = \left\{P \{\omega : c_T (\omega, \mathbf{h}) > c_{T,0}, \omega \in \Omega\} : \mathbf{h} \in \hat{\mathbf{h}}_\alpha\right\} \tag{25}$$

or

Table III. Fuzzy probability of alternative costs

Probability of occurrence of alternative				
lp.	Task	Degree of membership	p^-	p^+
	P_2	$\alpha = 0$	0.35	0.55
		$\alpha = 1/3$	0.3889	0.5222
		$\alpha = 2/3$	0.4407	0.4944
		$\alpha = 1$	0.4667	0.4667
	P_5	$\alpha = 0$	0.15	0.35
		$\alpha = 1/3$	0.1778	0.3111
		$\alpha = 2/3$	0.2148	0.2722
		$\alpha = 1$	0.2333	0.2333

$$\hat{R}_\alpha\left(c_{T,0}\right) = \left\{1 - \Phi_{c_T}\left(c_{T,0}, \mathbf{h}\right) : \mathbf{h} \in \hat{\mathbf{h}}_\alpha\right\} \tag{26}$$

where

$$\Phi_{c_T}\left(c_{T,0}, \mathbf{h}\right) = P\left\{\omega \in \Omega {:} c_T\left(\omega, \mathbf{h}\right) \le c_{T,0}\right\} \tag{27}$$

Fuzzy membership function $\mu\left(x|R_F\left(c_T\right)\right)$ of the risk of cost $R_F\left(c_T\right)$ can be described using the following formula:

$$\mu\left(x|R_F\left(c_T\right)\right) = sup\left\{\alpha : x \in \hat{R}_\alpha\left(c_{T,0}\right)\right\} \tag{28}$$

6. Approximate algorithm of calculation of fuzzy probability

We can find the approximate values of the fuzzy set$R_F\left(c_T\right)$ using alpha cut method and the formula (26).

1) For the discrete values $0 \le \alpha_1 \le \alpha_2 \le ... \le \alpha_k \le 1$ calculate alpha cut of the uncertain parameters $\hat{\mathbf{h}}_{\alpha_1}, \hat{\mathbf{h}}_{\alpha_2}, \ldots, \hat{\mathbf{h}}_{\alpha_k}$.

2) Divide the intervals $\hat{h}_{\alpha_i,1}, \hat{h}_{\alpha_i,2}, ..., \hat{h}_{\alpha_i,m}$ into k parts.

3) For each combination of the parameters $\left(h_{\alpha_i,1,j_1}, h_{\alpha_i,2,j_2}, ..., h_{\alpha_i,m,j_m}\right) = \mathbf{h}_{\alpha_i,j_1,j_2,...,j_m}$ calculate the cumulative distribution function $\Phi_{c_T}\left(c, \mathbf{h}_{\alpha_i,j_1,j_2,...,j_m}\right)$. The approximate value of the alpha cut $\hat{R}_{\alpha_i}\left(c_{T,0}\right)$ can be calculated. In the following way.

Table IV. Fuzzy costs

Alpha	**Task**	**min**		**mid**		**max**	
		c_o^-	c_o^+	c_m^-	c_m^+	c_p^-	c_p^+
$\alpha = 0$	**P0**	184,88	228,38	206,63	250,13	250,13	293,63
$\alpha = 1/3$		190,91	219,91	212,66	241,66	254,95	283,95
$\alpha = 2/3$		196,95	211,45	218,70	233,20	259,78	274,28
$\alpha = 1$		202,99	202,99	224,74	224,74	264,61	264,61
$\alpha = 0$	**P1**	112,20	125,40	125,40	151,80	151,80	178,20
$\alpha = 1/3$		114,40	123,20	129,07	146,67	154,73	172,33
$\alpha = 2/3$		116,60	121,00	132,73	141,53	157,66	166,46
$\alpha = 1$		118,80	118,80	136,40	136,40	160,59	160,59
$\alpha = 0$	**P2**	159,55	197,09	178,32	215,86	215,86	253,40
$\alpha = 1/3$		164,76	189,78	182,48	207,51	221,07	246,09
$\alpha = 2/3$		169,97	182,48	186,65	199,16	226,28	238,79
$\alpha = 1$		175,18	175,18	190,82	190,82	231,49	231,49
	P3	420,00	420,00	420,00	420,00	420,00	420,00
$\alpha = 0$	**P4**	222,45	274,79	248,62	300,96	327,13	379,47
$\alpha = 1/3$		232,62	267,51	258,79	293,68	334,39	369,26
$\alpha = 2/3$		242,79	260,23	268,96	286,40	341,66	359,05
$\alpha = 1$		252,96	252,96	279,13	279,13	348,92	348,85
$\alpha = 0$	**P5**	36,72	41,04	41,04	45,36	45,36	49,68
$\alpha = 1/3$		37,44	40,32	41,76	44,64	46,08	48,96
$\alpha = 2/3$		38,16	39,60	42,48	43,92	46,80	48,24
$\alpha = 1$		38,88	38,88	43,20	43,20	47,52	47,52
$\alpha = 0$	**P6**	93,98	119,04	119,04	131,57	144,10	169,16
$\alpha = 1/3$		98,84	115,55	121,12	129,48	146,88	163,58
$\alpha = 2/3$		103,71	112,07	123,21	127,39	149,66	158,01
$\alpha = 1$		108,58	108,58	125,30	125,30	152,44	152,44

$$R_{\alpha_i}^{-}\left(c_{T,0}\right) = \tag{29}$$

$$= min\left\{1 - \Phi_{c_T}\left(c_{T,0}, \mathbf{h}_{\alpha_i, j_1, j_2, \ldots, j_m}\right) : j_1, \ldots, j_m \in \{1, \ldots, k\}\right\} \tag{30}$$

$$R_{\alpha_i}^{+}\left(c_{T,0}\right) = \tag{31}$$

$$= max\left\{ 1 - \Phi_{c_T}\left(c_{T,0}, \mathbf{h}_{\alpha_i, j_1, j_2, ..., j_m}\right) : j_1, ..., j_m \in \{ 1, ..., k\}\right\}, \quad (32)$$

4) Approximate value of the fuzzy membership function $\mu\left(x|R_F\left(c_T\right)\right)$ is given by the following formula

$$\mu\left(x|R_F\left(c_{T,0}\right)\right) = sup\left\{\alpha_i : x \in \hat{R}_{\alpha_i}\left(c_{T,0}\right)\right\}, \quad (33)$$

7. Computer implementation of the algorithm

Algorithm which was described above was implemented in C++ language and can be run on Linux and Windows. To generation of random numbers GSL library was applied.

The models can be described by using BPFPRAL language (Bętkowski Pownuk Fuzzy Probability Risk Analysis Language) [5] . As and example below is show the code of simulator which is shown on the Fig. 1

```
Node
NumberOfNode 0, NumberOfChildren 2, Children 1 3, Probability 0.415,
IntervalProbability 0.088, xMinMin 198.766, xiMnMax 206.016, xMidMin
215.688, xMidMax 219.313, xMaxMin 231.391, xMaxMax 238.641, ProbabilityGrids 3
End

Node
NumberOfNode 1, NumberOfChildren 1, Children 2, xMinMin 125.761, xMinMax
130.161, xMidMin 133.830, xMidMax 138.230, xMaxMin 147.030, xMaxMax 153.63
End

Node
NumberOfNode 2, NumberOfChildren 1, Children 4, xMinMin 171.533, xMinMax
177.789, xMidMin 186.136, xMidMax 189.264, xMaxMin 206.983, xMaxMax 213.24
End

Node
PointValue, NumberOfNode 3, NumberOfChildren 1, Children 4, xMinMin 420.0,
xMinMax 420.0, xMidMin 420.0, xMidMax 420.0, xMaxMin 420.0, xMaxMax 420.0,
NumberOfGrid 1
End

Node
NumberOfNode 4, NumberOfChildren 2, Children 5 6, Probability 0.224,
IntervalProbability 0.088, xMinMin 239.159, xMinMax 247.882, xMidMin
252.244, xMidMax 260.967, xMaxMin 282.863, MaxMax 295.948, NumberOfGrid 2,
ProbabilityGrids 3
End

Node
NumberOfNode 5, NumberOfChildren 1, Children 6, xMinMin 38.52, xMinMax
40.68, xMidMin 42.84, xMidMax 44.28, xMaxMin 47.40, xMaxMax 48.84
End
```

```
Node
NumberOfNode 6, xMinMin 121.123, xMinMax 125.3, xMidMin 126.344, xMidMax
130.521, xMaxMin 140.267, xMaxMax 146.532, NumberOfGrid 2
End

Results
Xmin 820, Xmax 1120, NumberOfSimulations 10000, NumberOfClasses 20,
NumberOfGrid 2, DistributionType 2
End
```

In presented example only one alpha cut was described. In order to get full description of fuzzy probability model it is necessary to repeat these calculations for each alpha cut.

In the program we can define the upper and lower bounds of c_o, c_m, c_p in the following way:

$$cMinMin \leq c_o \leq cMinMax \tag{34}$$

$$cMidMin \leq c_m \leq cMidMax \tag{35}$$

$$cMaxMin \leq c_p \leq cMaxMax \tag{36}$$

The meaning of other instructions is explaind in the BPFPRAL user manual.

8. Numerical results of the calculations

For the example which is shown on the Fig. 1 and is also described in the BPFPRAL language above. In the numerical experiment 10000 Monte Carlo simulations was used for each combination of uncertain parameters in each alpha cut. Extreme values of risk and probability density function of cost were calculated by using 262144 combinations of uncertain parameters.

The envelopes of the risk curves for particular alpha level equal to 0.33 are shown below.

Now we can show the shape of fuzzy risk surfaces for particular alpha levels on the Fig. 4, 5, 8 .

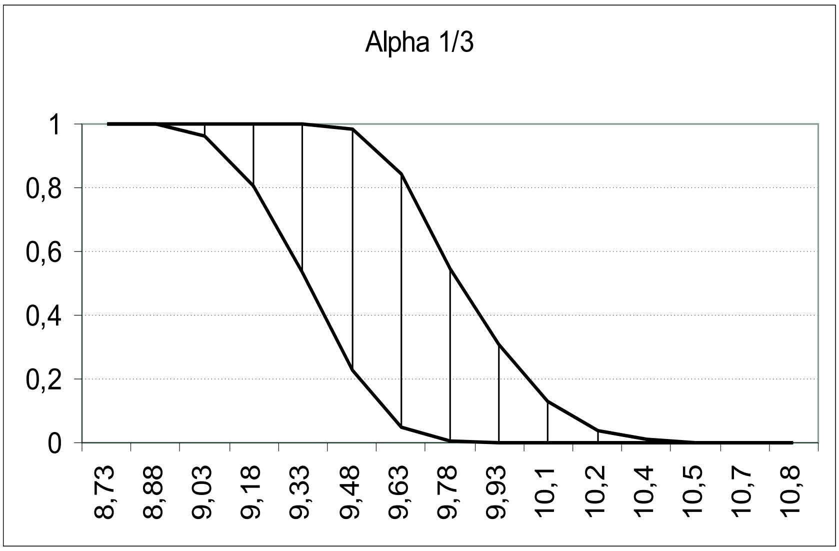

Figure 3. Uncertain risc curve for $\alpha = 1/3$

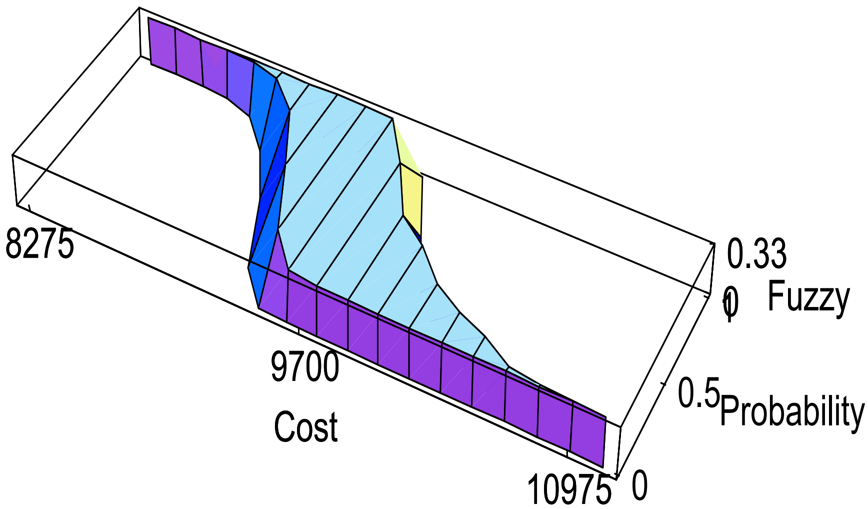

Figure 4. Fuzzy probability surface for $\alpha = 1/3$

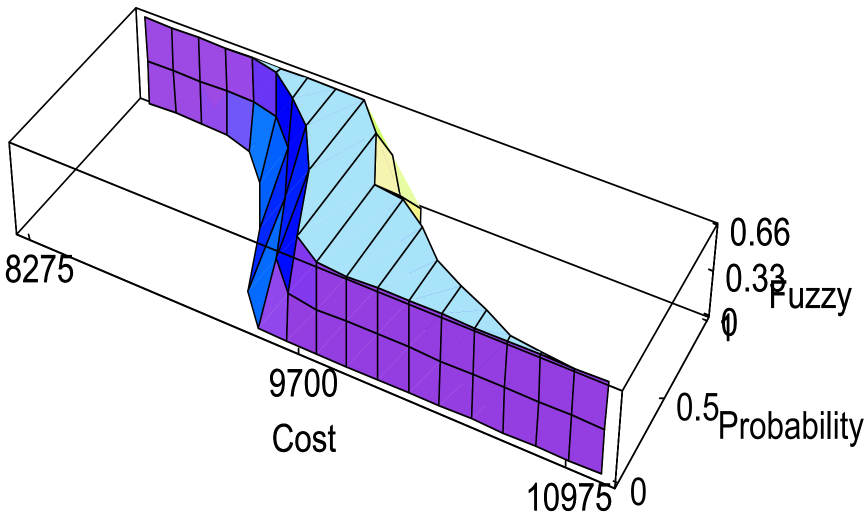

Figure 5. Fuzzy probability surface for $\alpha = 2/3$

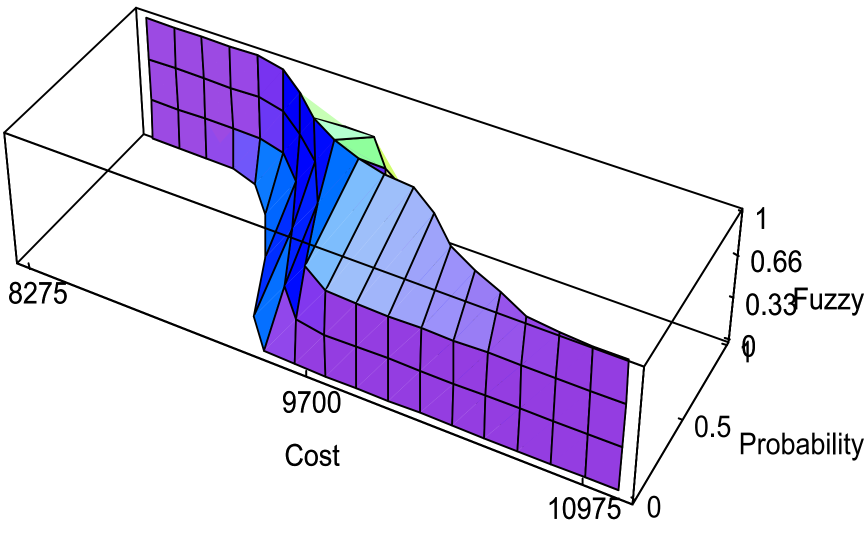

Figure 6. Fuzzy probability surface for $\alpha = 1$

Table V. Numerical results

Cost	**Probability**						
	$\alpha = 0$		$\alpha = 1/3$		$\alpha = 2/3$		$\alpha = 1$
	Min	Max	Min	Max	Min	Max	
8350 - 8500	1	1	1	1	1	1	1
8500 - 8650	1	1	1	1	1	1	1
8650 - 8800	0,989	1	1	1	1	1	1
8800 - 8950	0,941	1	0,997	1	0,997	1	0,997
8950 - 9100	0,784	1	0,954	1	0,964	1	0,964
9100 - 9250	0,528	1	0,808	1	0,85	1	0,85
9250- 9400	0,172	1	0,515	1	0,719	1	0,719
9400 - 9550	0,006	1	0,179	1	0,481	0,989	0,634
9550 - 9700	0	0.992	0,019	0,99	0,102	0,824	0,574
9700 - 9850	0	0.927	0	0,78	0,007	0,531	0,531
9850 - 10000	0	0,691	0	0,514	0	0,514	0,514
10000 - 10150	0	0,406	0	0,406	0	0,406	0,406
10150 - 10300	0	0,259	0	0,259	0	0,259	0,259
10300 - 10450	0	0,176	0	0,176	0	0,176	0,176
10450 - 10600	0	0,111	0	0,111	0	0,111	0,111
10600 - 10750	0	0,031	0	0,031	0	0,031	0,031
10750 -10900	0	0,009	0	0,009	0	0,009	0,009
10900 - 11050	0	0	0	0	0	0	0
11050 - 11200	0	0	0	0	0	0	0

9. Conclusions

Presented method allows estimating the direct cost risk of civil engineering projects in the case when there are no credible data. In presented algorithm the costs can be deterministic, probabilistic, fuzzy number. It is also possible to take into account the cost which is modeled by probability density function with fuzzy parameters. Unfortunately, at this moment the computational complexity of the algorithm grows exponentially with respect to the number of the fuzzy parameters. The method shows the relation between the assumed maximal direct costs, the risk of overrun and the uncertainty of the statistical data.

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