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Adaptive Taylor Series and its Applications in the Interval Finite Element Method

Equation with uncertain parameters

$$ax = b \qquad x = \frac{b}{a}$$

Example

$$[1,2]x = [1,4]$$

$$x = ?$$

Algebraic solution

$$[1,2]x = [1,4]$$

$$x = [1,2]$$

because

$$[1,2][1,2] = [1,4]$$

Algebraic solution

$$[1,4]x=[1,4]$$

$$x=[1,1]=1$$

because

$$[1,4]\cdot 1=[1,4]$$

Algebraic solution

$$[1,8]x = [1,4]$$

$$x = ?$$

Algebraic solution do not have physical interpretation.

United solution set

$$[1,2]x=[1,4]$$

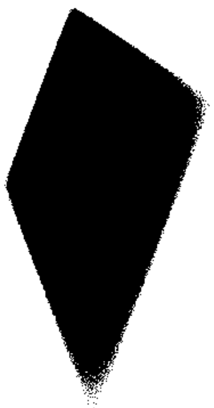
$$\mathbf{x}=\left[\frac{1}{2},4\right]$$

because

$$\mathbf{x}=\{x:ax=b,a\in[1,2],b\in[1,4]\}$$

Web applications (Java)

System of equations with the random parameters



Get results

2D histogram

```
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.1932731022155822E-5 5.194911066923414E-5 7.895001
0.0 0.0 0.0 0.0 7.737101589034872E-6 0.0013841742414225854 0.006519026903154
0.0 0.0 1.0538699307280152E-4 0.003945493224605476 0.022974849468541763 0.0
0.0 0.0 9.875857528289511E-4 0.016444498877347686 0.054662690397973834 0.0
0.0 0.0 2.5545969532309014E-5 0.004332269354041005 0.019751703778005277 0.0
0.0 0.0 0.0 1.643513766113938E-4 0.0032391725438280583 0.007954552490837484
0.0 0.0 0.0 0.0 1.7340807132858768E-4 8.798077021222204E-4 0.001127090431480
0.0 0.0 0.0 0.0 4.398643760530029E-7 2.3414319094513694E-5 2.365116914315761
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
```

Number of simulations: 88663693

1D histogram for x1

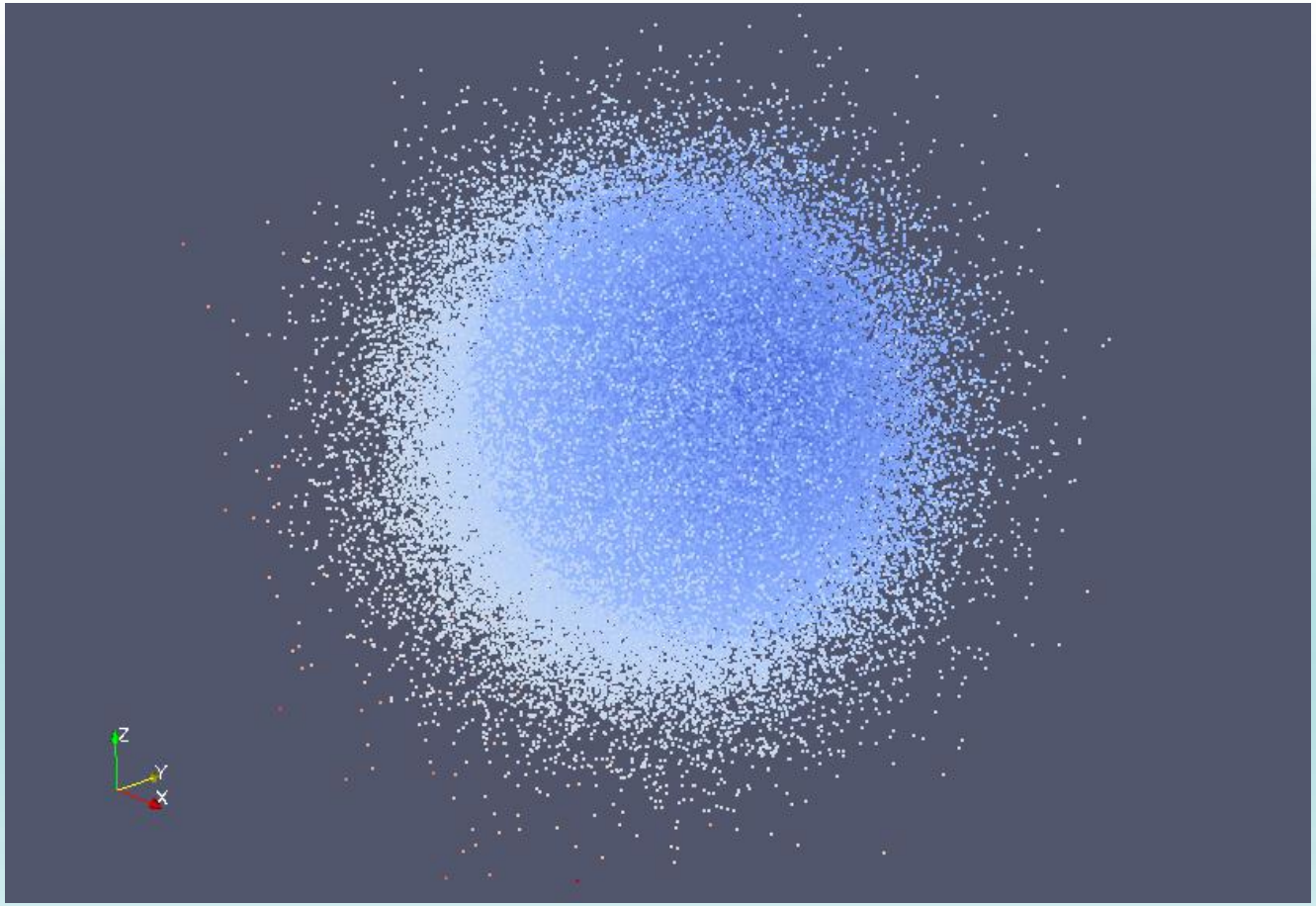
```
0.03795473531651789
0.0034054412779760935
5.0437781787411E-5
0.0
0.0
0.0
0.0
0.0
0.0
0.0
```

1D histogram for x2

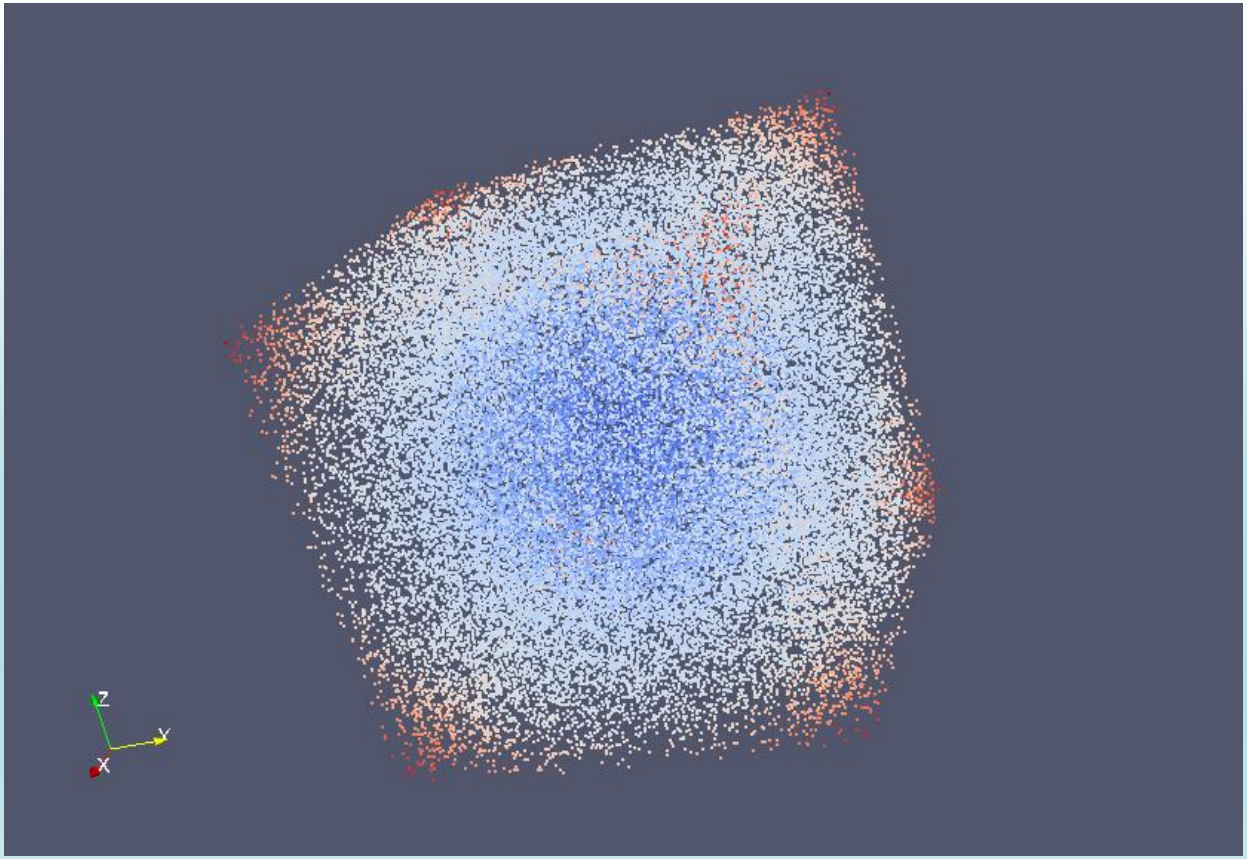
```
0.026894526037845052
```

Done | Internet | Protected Mode: On | 100%

Monte Carlo method 3D solution set



Monte Carlo method 3D solution set



Solution Set

$$\mathbf{u}_i(\mathbf{p}) = \{u_i(p) : A(p)u - f(p) = 0, p \in \mathbf{p}\}$$

$$[\underline{u}_i(\mathbf{p}), \overline{u}_i(\mathbf{p})] = \{u_i(p) : A(p)u - f(p) = 0, p \in \mathbf{p}\}$$

Monotonicity of the solution

- Monotone solution

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p_1 + p_2 \\ p_2 \end{bmatrix}$$

$$u_1 = \frac{p_1}{2} + p_2,\ u_2 = \frac{p_1}{2}$$

- Non-monotone solution

$$u^2 - p^4 = 0$$

$$u_1 = p^2, u_2 = -p^2$$

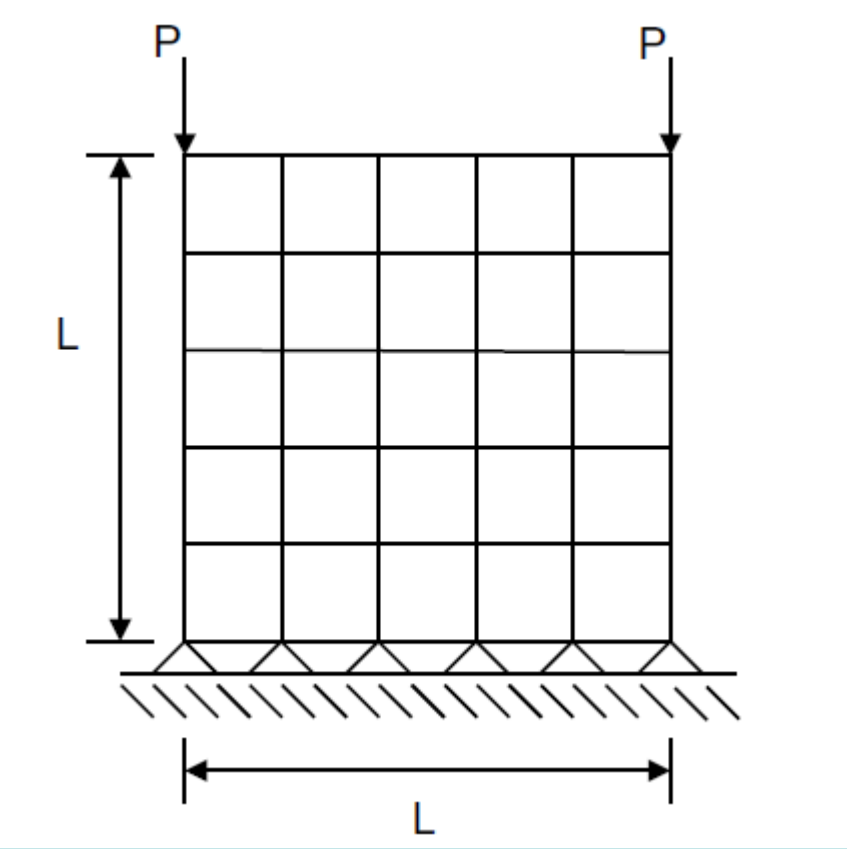
Interval solution for monotone functions (gradient descent method)

If $\frac{\partial u}{\partial p} \geq 0$ then $p^{min} = \underline{p}, p^{max} = \overline{p}$

If $\frac{\partial u}{\partial p} < 0$ then $p^{min} = \overline{p}, p^{max} = \underline{p}$

$$\underline{u} = u(\underline{p}^{min}), \overline{u} = u(\underline{p}^{max})$$

Monotonicity of the solution



Plane stress

$$\begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix} = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix},$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{bmatrix},$$

$$\begin{bmatrix} \partial/\partial x & 0 & \partial/\partial y \\ 0 & \partial/\partial y & \partial/\partial x \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Interval solution
Verification of the results by using search method with 3 intermediate points

Table VIII. Displacements in the case of 4 (2x2) finite elements and 5% uncertainty

	Combinatoric	Combinatoric	Gradient free	Gradient free		
DOF	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$	Error $\underline{u}$ %	Error $\overline{u}$ %
5	-8.770130E-23	1.113670E-22	-8.770130E-23	1.113670E-22	0.000000E+00	0.000000E+00
6	-4.404760E-07	-3.605710E-07	-4.404760E-07	-3.605710E-07	0.000000E+00	0.000000E+00
7	4.822140E-08	5.890760E-08	4.822140E-08	5.890760E-08	0.000000E+00	0.000000E+00
8	-6.087950E-07	-4.983560E-07	-6.087950E-07	-4.983560E-07	0.000000E+00	0.000000E+00
11	-5.890760E-08	-4.822140E-08	-5.890760E-08	-4.822140E-08	0.000000E+00	0.000000E+00
12	-6.087950E-07	-4.983560E-07	-6.087950E-07	-4.983560E-07	0.000000E+00	0.000000E+00
13	-1.851990E-23	2.469330E-22	-1.851990E-23	2.469330E-22	0.000000E+00	0.000000E+00
14	-3.540790E-07	-2.898470E-07	-3.540790E-07	-2.898470E-07	0.000000E+00	0.000000E+00
15	-4.815970E-07	-3.942320E-07	-4.815970E-07	-3.942320E-07	0.000000E+00	0.000000E+00
16	-1.840780E-06	-1.506850E-06	-1.840780E-06	-1.506850E-06	0.000000E+00	0.000000E+00
17	3.942320E-07	4.815970E-07	3.942320E-07	4.815970E-07	0.000000E+00	0.000000E+00
18	-1.840780E-06	-1.506850E-06	-1.840780E-06	-1.506850E-06	0.000000E+00	0.000000E+00

Interval solution

Table X. Displacements in the case of 9 (3x3) finite elements and 5% uncertainty

	Combinatoric		Gradient free			
DOF	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$	Error $\underline{u}$ %	Error $\overline{u}$ %
5	2.991120E-09	4.758950E-08	2.991120E-09	4.758950E-08	0.000000E+00	0.000000E+00
6	-3.401560E-07	-2.737550E-07	-3.401560E-07	-2.738240E-07	0.000000E+00	2.520502E-02
7	-2.700300E-08	3.178720E-08	-2.681040E-08	3.178720E-08	7.132541E-01	0.000000E+00
8	-4.259370E-07	-2.781400E-07	-4.259370E-07	-2.781400E-07	0.000000E+00	0.000000E+00
11	-4.758950E-08	-2.991120E-09	-4.758950E-08	-2.991120E-09	0.000000E+00	0.000000E+00
12	-3.401560E-07	-2.737550E-07	-3.401560E-07	-2.738240E-07	0.000000E+00	2.520502E-02
15	-3.178720E-08	2.700300E-08	-3.178720E-08	2.681040E-08	0.000000E+00	7.132541E-01
16	-4.259370E-07	-2.781400E-07	-4.259370E-07	-2.781400E-07	0.000000E+00	0.000000E+00
17	-8.584880E-09	1.665680E-07	-8.584880E-09	1.665680E-07	0.000000E+00	0.000000E+00
18	-5.498820E-07	-4.324890E-07	-5.498820E-07	-4.324890E-07	0.000000E+00	0.000000E+00

Interval solution

Table XI. Combinations of parameters which correspond to the interval displacements 9 elements (3x3) and 5% uncertainty

	Combinatoric	Combinatoric	Gradient free	Gradient free		
DOF	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$
5	0,1,1,0,1,0,1,1,1,0,1	1,0,0,1,0,1,0,0,0,1,0	0,1,1,0,1,0,1,1,1,0,1	1,0,0,1,0,1,0,0,0,1,0	0	0
6	0,0,0,0,1,0,1,1,1,0,1	1,1,1,[0],0,1,0,0,0,1,0	0,0,0,0,1,0,1,1,1,0,1	1,1,1,[1],0,1,0,0,0,1,0	0	1
7	0,0,1,0,1,0,1,1,1,0,1	1,[0],0,1,0,1,0,0,0,1,0	0,[1],1,0,1,0,1,1,1,0,1	1,0,0,1,0,1,0,0,0,1,0	1	0
8	0,0,1,1,0,1,0,0,0,0,1	1,1,0,0,1,0,1,1,1,1,0	0,0,1,1,0,1,0,0,0,0,1	1,1,0,0,1,0,1,1,1,1,0	0	0
11	0,0,1,1,0,1,0,0,0,0,1	1,1,0,0,1,0,1,1,1,1,0	0,0,1,1,0,1,0,0,0,0,1	1,1,0,0,1,0,1,1,1,1,0	0	0
12	0,0,0,0,1,0,1,1,1,1,0	1,1,1,1,0,[0],0,0,0,0,1	0,0,0,0,1,0,1,1,1,1,0	1,1,1,1,0,[1],0,0,0,0,1	0	1
15	0,0,1,1,0,1,0,0,0,0,1	1,[0],0,0,1,0,1,1,1,1,0	0,0,1,1,0,1,0,0,0,0,1	1,[1],0,0,1,0,1,1,1,1,0	0	1
16	1,0,0,1,0,1,0,0,0,1,0	0,1,1,0,1,0,1,1,1,0,1	1,0,0,1,0,1,0,0,0,1,0	0,1,1,0,1,0,1,1,1,0,1	0	0
17	0,1,1,1,1,1,0,1,0,0,1	1,0,0,0,0,0,1,0,1,1,0	0,1,1,1,1,1,0,1,0,0,1	1,0,0,0,0,0,1,0,1,1,0	0	0
18	0,0,0,0,0,0,1,1,1,0,1	1,1,1,1,1,1,0,0,0,1,0	0,0,0,0,0,0,1,1,1,0,1	1,1,1,1,1,1,0,0,0,1,0	0	0

Interval solution

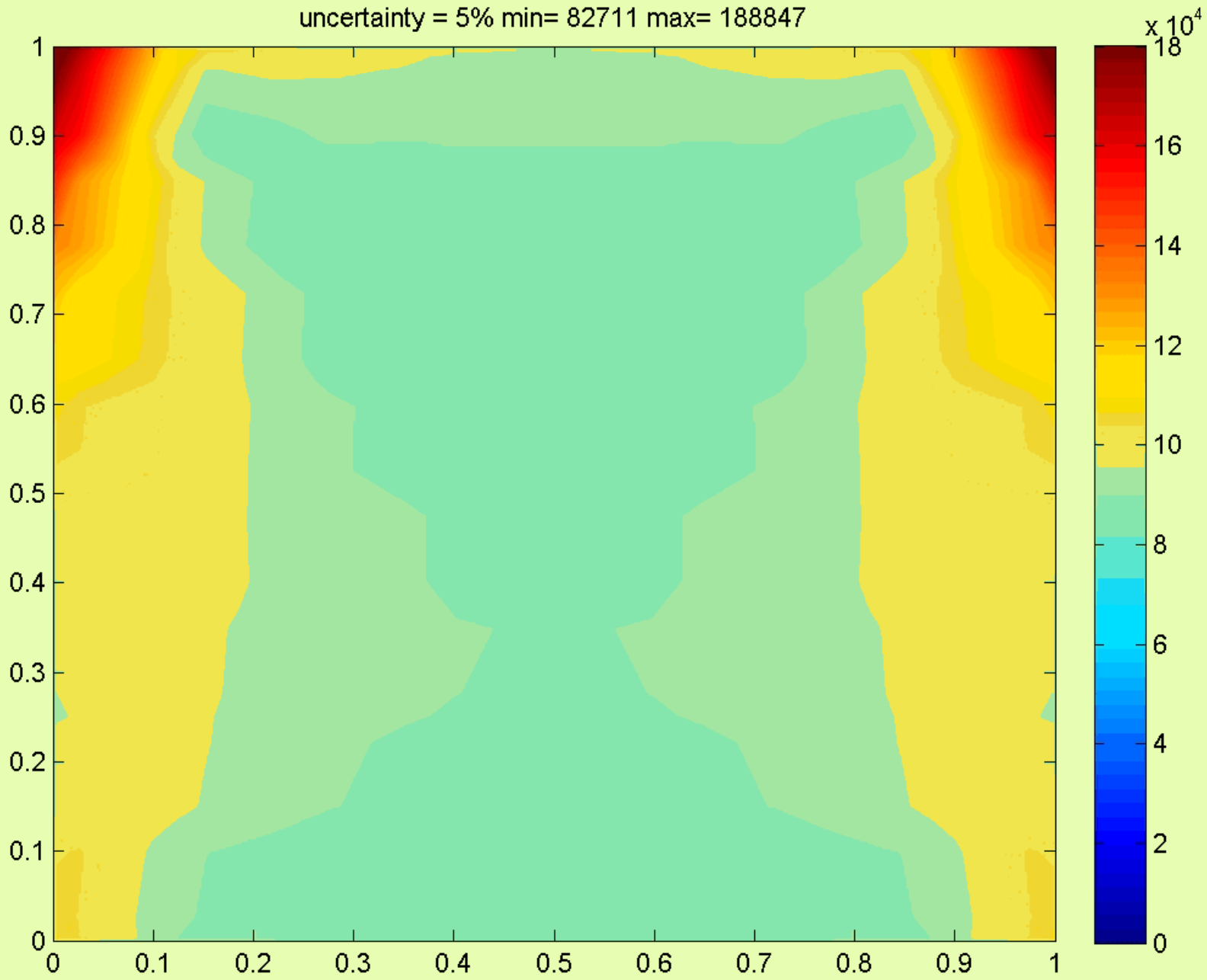
Table XII. Interval displacements for 10% uncertainty and 9 finite elements

DOF	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$	Error $\underline{u}$ %	Error $\overline{u}$ %
5	-1.934900E-08	7.022680E-08	-1.934900E-08	7.022470E-08	0.000000E+00	2.990311E-03
6	-3.796920E-07	-2.451030E-07	-3.796920E-07	-2.458330E-07	0.000000E+00	2.978340E-01
7	-5.781790E-08	6.089000E-08	-5.677190E-08	6.089000E-08	1.809128E+00	0.000000E+00
8	-5.089660E-07	-2.112750E-07	-5.089660E-07	-2.112750E-07	0.000000E+00	0.000000E+00
11	-7.022680E-08	1.934900E-08	-7.022470E-08	1.934900E-08	2.990311E-03	0.000000E+00
12	-3.796920E-07	-2.451030E-07	-3.796920E-07	-2.458330E-07	0.000000E+00	2.978340E-01
15	-6.089000E-08	5.781790E-08	-6.089000E-08	5.677190E-08	0.000000E+00	1.809128E+00
16	-5.089660E-07	-2.112750E-07	-5.089660E-07	-2.112750E-07	0.000000E+00	0.000000E+00
17	-9.469690E-08	2.590120E-07	-9.315200E-08	2.590120E-07	1.631416E+00	0.000000E+00
18	-6.187590E-07	-3.820950E-07	-6.187590E-07	-3.820950E-07	0.000000E+00	0.000000E+00

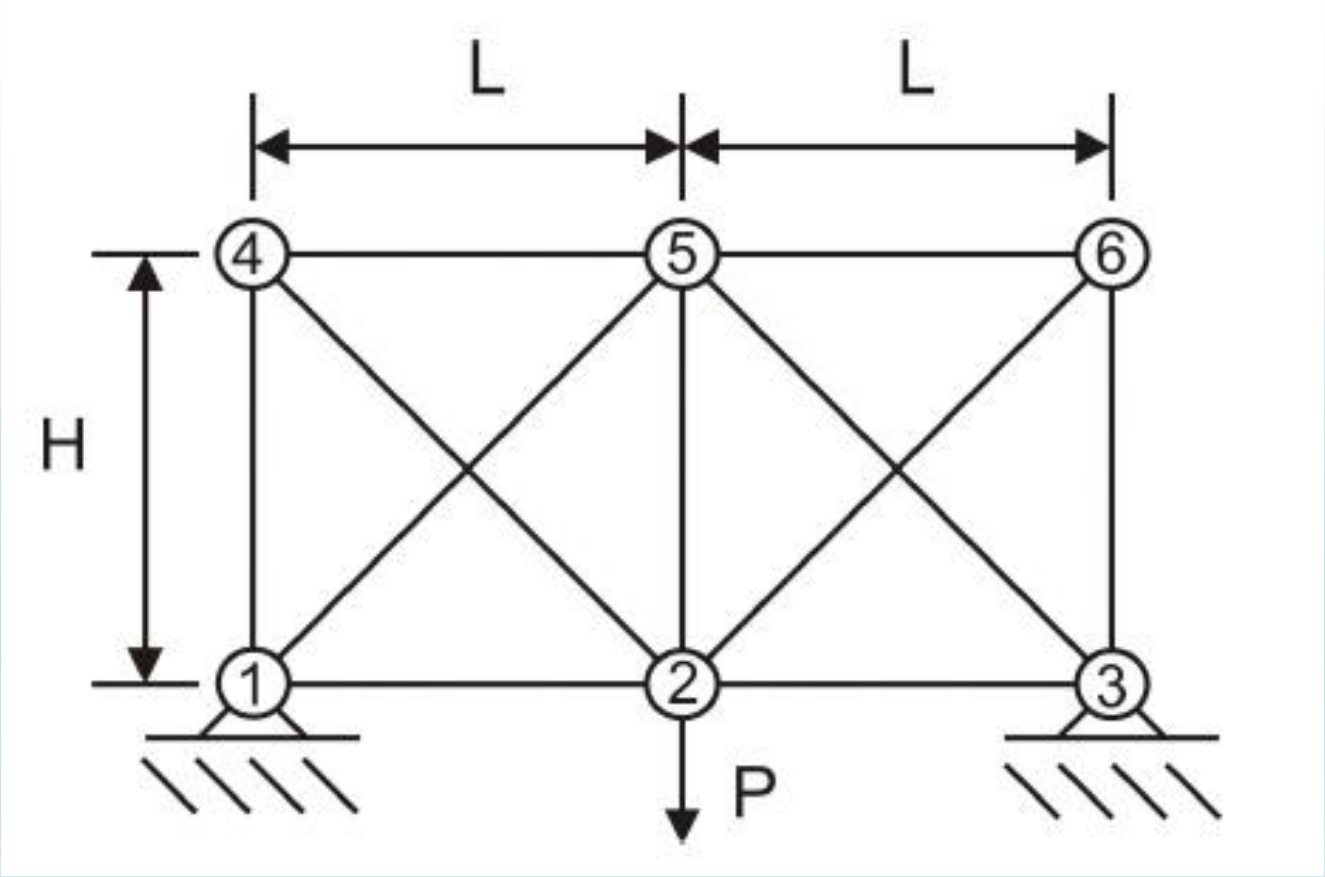
Interval solution

Table XV. Interval displacements 10% uncertainty

$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$		
0,1,1,0,1,0,1,1,1,0,1	1,0,0,1,0,1,[1],0,0,1,0	0,1,1,0,1,0,1,1,1,0,1	1,0,0,1,0,1,[0],0,0,1,0	0	1
0,0,0,0,1,0,1,1,1,0,1	1,1,1,[0],0,1,0,0,0,1,0	0,0,0,0,1,0,1,1,1,0,1	1,1,1,[1],0,1,0,0,0,1,0	0	1
0,0,1,0,1,0,1,1,1,0,1	1,[0],0,1,0,1,0,0,0,1,0	0,[1],1,0,1,0,1,1,1,0,1	1,0,0,1,0,1,0,0,0,1,0	1	0
0,0,1,1,0,1,0,0,0,0,1	1,1,0,1,1,0,1,1,1,1,0	0,0,1,1,0,1,0,0,0,0,1	1,1,0,1,1,0,1,1,1,1,0	0	0
0,0,1,1,0,1,0,0,1,0,1	1,1,0,0,1,0,1,1,[1],1,0	0,0,1,1,0,1,0,0,[0],0,1	1,1,0,0,1,0,1,1,1,1,0	1	0
0,0,0,0,1,0,1,1,1,1,0	1,1,1,1,0,[0],0,0,0,0,1	0,0,0,0,1,0,1,1,1,1,0	1,1,1,1,0,[1],0,0,0,0,1	0	1
0,0,1,1,0,1,0,0,0,0,1	1,[0],0,0,1,0,1,1,1,1,0	0,0,1,1,0,1,0,0,0,0,1	1,[1],0,0,1,0,1,1,1,1,0	0	1
1,0,0,1,0,1,0,0,0,1,0	0,1,1,0,1,1,1,1,1,0,1	1,0,0,1,0,1,0,0,0,1,0	0,1,1,0,1,1,1,1,1,0,1	0	0
0,0,1,0,1,1,0,1,0,0,1	1,[0],0,[0],0,0,1,0,1,1,0	0,[1],1,[1],1,1,0,1,0,0,1	1,0,0,0,0,0,1,0,1,1,0	2	0
0,0,0,0,0,0,1,1,1,0,1	1,1,1,1,1,1,0,0,0,1,0	0,0,0,0,0,0,1,1,1,0,1	1,1,1,1,1,1,0,0,0,1,0	0	0
0,1,1,1,1,1,0,0,0,0,1	1,0,0,0,0,0,1,1,1,1,0	0,1,1,1,1,1,0,0,0,0,1	1,0,0,0,0,0,1,1,1,1,0	0	0
0,0,1,0,0,1,0,0,0,0,1	1,1,0,1,1,0,1,1,1,1,0	0,0,1,0,0,1,0,0,0,0,1	1,1,0,1,1,0,1,1,1,1,0	0	0
0,0,1,0,0,0,1,0,1,0,1	1,[0],0,1,1,[0],0,1,0,1,0	0,0,1,0,0,0,1,0,1,0,1	1,[1],0,1,1,[1],0,1,0,1,0	0	2


uncertainty = 5% min= 82711 max= 188847
x 10^4
18
16
14
12
10
8
6
4
2
0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Truss example



Truss structure

$$\frac{d}{dx}\left(EA\,\frac{du}{dx} \right) + n = 0$$

$$k^{(1)} = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow K^{(1)} = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \end{bmatrix} \quad \begin{aligned} s &= \sin\beta \\ c &= \cos\beta \end{aligned}$$

Interval solution
Verifications of the results by using search method with 5 intermediate points

Table XVI. Displacement in 11 bar truss 5% uncertainty

	Combinatoric		Gradient free		Error	
DOF	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$	$\underline{u}$ %	$\overline{u}$ %
3	-1.538050E-04	1.538050E-04	-1.388490E-04	1.420150E-04	9.724001E+00	7.665551E+00
4	-1.652290E-02	-1.352560E-02	-1.652290E-02	-1.352560E-02	0.000000E+00	0.000000E+00
7	2.440140E-03	3.537200E-03	2.440140E-03	3.536200E-03	0.000000E+00	2.827095E-02
8	-8.664030E-04	-6.318360E-04	-8.664030E-04	-6.318520E-04	0.000000E+00	2.532303E-03
9	-3.510260E-04	3.510260E-04	-3.172980E-04	3.506350E-04	9.608405E+00	1.113878E-01
10	-1.409620E-02	-1.140120E-02	-1.409620E-02	-1.140120E-02	0.000000E+00	0.000000E+00
11	-3.537200E-03	-2.440140E-03	-3.537200E-03	-2.440960E-03	0.000000E+00	3.360463E-02
12	-8.664030E-04	-6.318360E-04	-8.664030E-04	-6.318520E-04	0.000000E+00	2.532303E-03

Interval solution

Table XVIII. Binary code of the interval displacements in 11 bar truss for 5% uncertainty

	Combinatoric		Gradient free	
DOF	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$
3	0,0,1,0,1,0,0,1,0,0,1,0	[0],[0],0,1,0,[0],1,0,1,1,0,[0]	0,0,1,0,1,[1],0,1,0,0,1,[1]	[1],[1],0,1,0,0,1,0,1,1,0,0
4	0,0,0,0,0,0,0,0,0,0,0,0	1,1,1,1,1,1,1,1,1,1,1,1	0,0,0,0,0,0,0,0,0,0,0,0	1,1,1,1,1,1,1,1,1,1,1,1
7	0,0,1,1,0,1,1,0,0,1,1,1	[0],[0],0,0,1,0,0,1,1,0,0,0	0,0,1,1,0,1,1,0,0,1,1,1	[1],[1],0,0,1,0,0,1,1,0,0,0
8	0,0,1,1,0,0,1,1,0,1,0,0	[0],[0],0,0,1,1,0,0,1,0,1,1	0,0,1,1,0,0,1,1,0,1,0,0	[1],[1],0,0,1,1,0,0,1,0,1,1
9	1,1,0,1,0,1,1,0,0,1,1,0	[1],[1],1,0,1,[1],0,1,1,0,0,[0]	[0],[0],0,1,0,1,1,0,0,1,1,[1]	1,1,1,0,1,[0],0,1,1,0,0,0
10	1,0,0,0,0,1,0,0,0,0,0,0	[1],1,1,1,1,0,1,1,1,1,1,1	[0],0,0,0,0,1,0,0,0,0,0,0	1,1,1,1,1,0,1,1,1,1,1,1
11	0,0,0,0,0,0,1,0,0,1,1,0	[0],[0],1,1,1,1,0,1,1,0,0,1	0,0,0,0,0,0,1,0,0,1,1,0	[1],[1],1,1,1,1,0,1,1,0,0,1
12	0,0,1,1,1,0,0,1,0,1,0,0	[0],[0],0,0,0,1,1,0,1,0,1,1	0,0,1,1,1,0,0,1,0,1,0,0	[1],[1],0,0,0,1,1,0,1,0,1,1

Interval solution

Table XVII. Interval displacement for 10%

	Combinatoric		Gradient free		Error	
DOF	$\underline{u}$	$\overline{u}$	$\underline{u}$	$\overline{u}$	$\underline{u}$ %	$\overline{u}$ %
3	-3.372950E-04	3.372950E-04	-2.341290E-04	3.372950E-04	3.058628E+01	0.000000E+00
4	-1.827140E-02	-1.223130E-02	-1.827140E-02	-1.223130E-02	0.000000E+00	0.000000E+00
7	1.960710E-03	4.171630E-03	1.963690E-03	4.171630E-03	1.519858E-01	0.000000E+00
8	-1.011620E-03	-5.371630E-04	-1.011620E-03	-5.372160E-04	0.000000E+00	9.866651E-05
9	-7.417680E-04	7.417680E-04	-7.401790E-04	6.520410E-04	2.142179E-01	1.209637E-01
10	-1.566970E-02	-1.023890E-02	-1.566970E-02	-1.023890E-02	0.000000E+00	0.000000E+00
11	-4.171630E-03	-1.960710E-03	-4.167180E-03	-1.960710E-03	1.066729E-01	0.000000E+00
12	-1.011620E-03	-5.371630E-04	-1.011620E-03	-5.372160E-04	0.000000E+00	9.866651E-05

Verification of the monotonicity by using second order monotonicity test

Table XV. Derivative of the displacement u_5 with respect to Young's modulus and point load

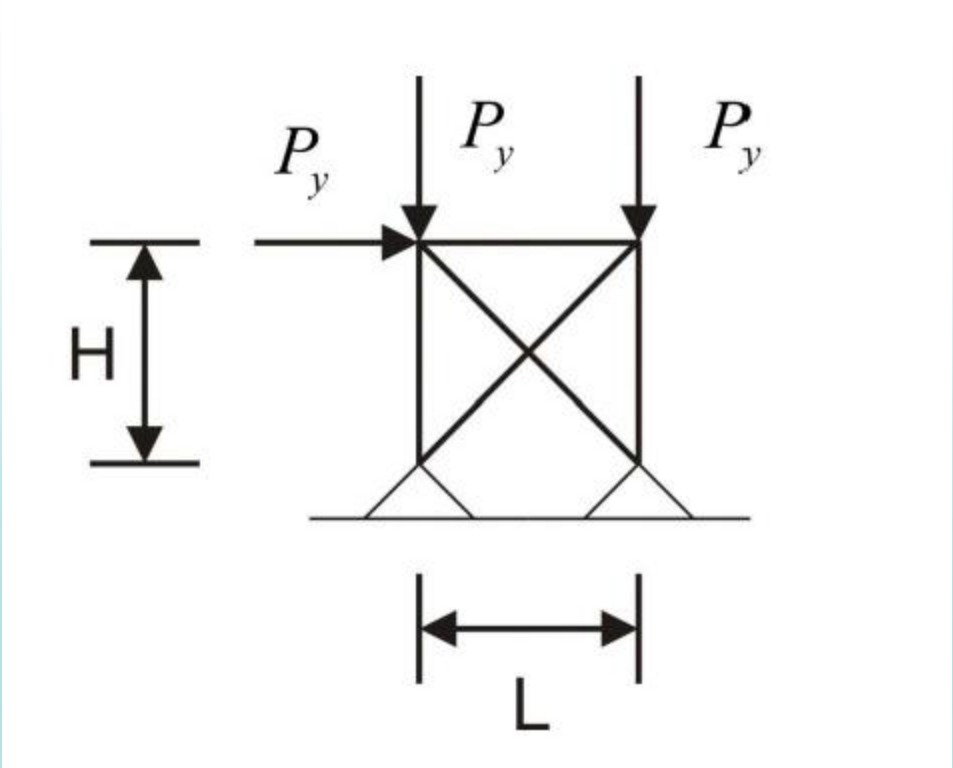
Derivative	Min	Max	Sign
$\frac{\partial u_5}{\partial E_1}$	-3.26934E-16	2.508830E-16	?
$\frac{\partial u_5}{\partial E_2}$	-3.26934E-16	2.50883E-16	?
$\frac{\partial u_5}{\partial E_3}$	-1.92976E-15	-1.24505E-15	−
$\frac{\partial u_5}{\partial E_4}$	1.24505E-15	1.92976E-15	+
$\frac{\partial u_5}{\partial E_5}$	-4.97695E-16	-3.01577E-16	−
$\frac{\partial u_5}{\partial E_6}$	-1.15435E-16	8.06621E-17	?
$\frac{\partial u_5}{\partial E_7}$	3.01578E-16	4.97696E-16	+
$\frac{\partial u_5}{\partial E_8}$	-2.6533E-15	-1.76977E-15	−
$\frac{\partial u_5}{\partial E_9}$	2.53431E-15	3.60947E-15	+
$\frac{\partial u_5}{\partial E_{10}}$	1.76977E-15	2.6533E-15	+
$\frac{\partial u_5}{\partial E_{11}}$	-3.61005E-15	-2.53463E-15	−
$\frac{\partial u_5}{\partial E_P}$	-9.13142E-09	9.93587E-09	?

More examples

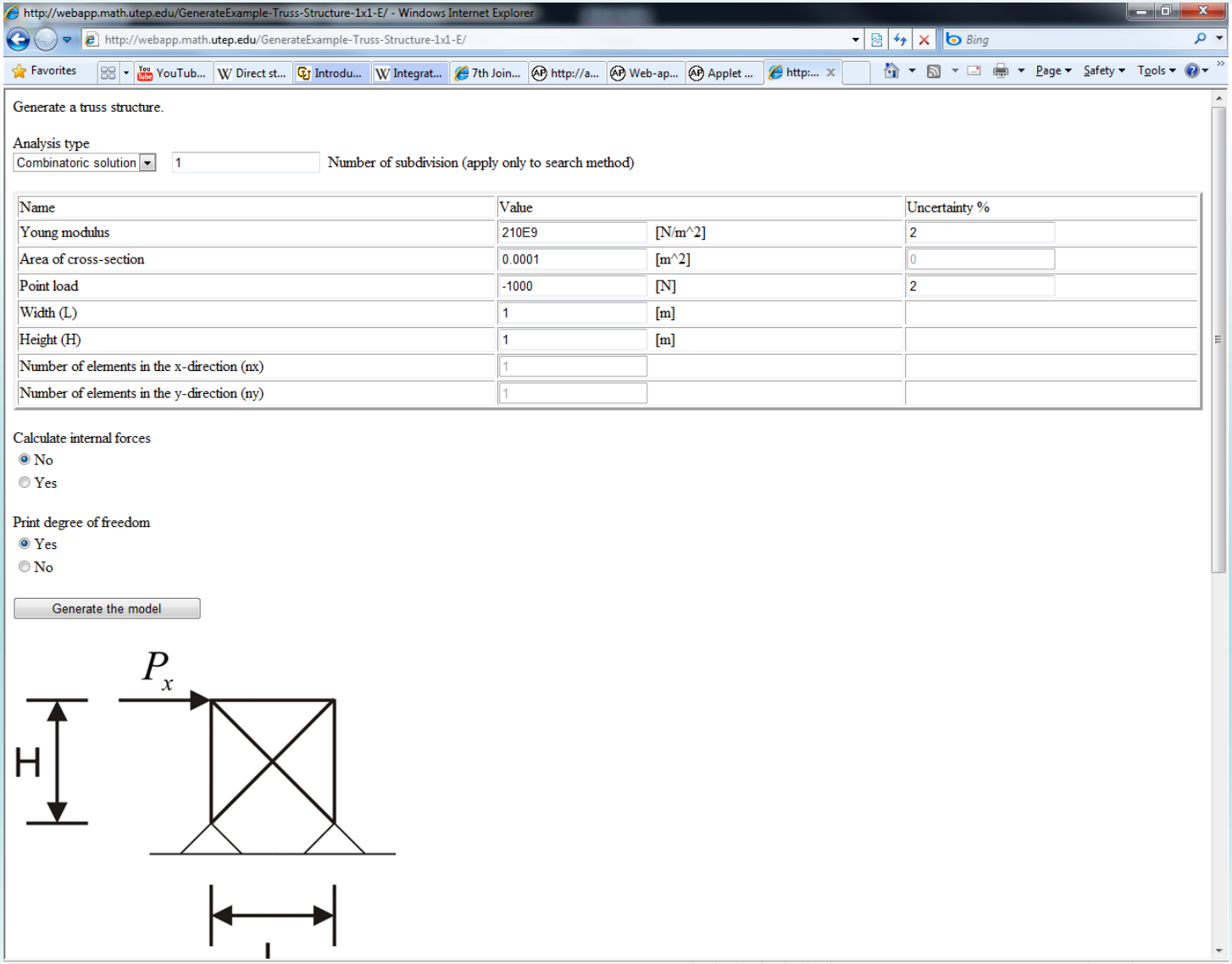
A. Pownuk, Monotonicity of the solution of the interval equations of structural mechanics - list of examples,
The University of Texas at El Paso, Department of Mathematical Sciences Research Reports Series
Texas Research Report No. 2009-01, El Paso, Texas, USA [Download]

817 pages

Example



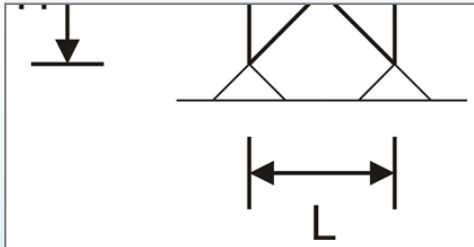
Automatic generation of examples



Scripting language

http://webapp.math.utep.edu/GenerateExample-Truss-Structure-1x1-E/default.aspx - Windows Internet Explorer

http://webapp.math.utep.edu/GenerateExample-Truss-Structure-1x1-E/default.aspx




```
# *****************
# Preprocessing
# *****************

start_clock

analysis_type linear_static_interval_combinatoric

# Parameter list
parameter 1 [2.079E+11, 2.121E+11]# Young modulus in the element 1
parameter 2 [2.079E+11, 2.121E+11]# Young modulus in the element 2
parameter 3 [2.079E+11, 2.121E+11]# Young modulus in the element 3
parameter 4 [2.079E+11, 2.121E+11]# Young modulus in the element 4
parameter 5 [2.079E+11, 2.121E+11]# Young modulus in the element 5
parameter 6 0.0001 # Area of cross-section in the element 1
parameter 7 0.0001 # Area of cross-section in the element 2
parameter 8 0.0001 # Area of cross-section in the element 3
parameter 9 0.0001 # Area of cross-section in the element 4
parameter 10 0.0001 # Area of cross-section in the element 5
parameter 11 [-1010, -990] # Point load on node 3 x-direction

# The list of points

point 1 x 0 y 0
point 2 x 1 y 0
point 3 x 0 y 1
point 4 x 1 y 1

# The list of lines

line 1 points 1 3 parameters 1 6
line 2 points 1 4 parameters 2 7
line 3 points 2 3 parameters 3 8
line 4 points 4 3 parameters 4 9
line 5 points 2 4 parameters 5 10

# The list of loads

load point_load point 3 fx 11
```

Done

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Results of the calculations

```
# ********************
# Postprocessing
# ********************

print_interval_displacements
print_Jacobian
print_Jacobian_binary
print_parameters
print_dof all
print_number_DOF
print_time
print_number_of_simulations
```

2D interval solution

```
# ********************
# Postprocessing
# ********************

print_interval_displacements
print_interval_stress
print_interval_Mises_stress
print_interval_Mises_stress_to_matlab
print_interval_stress_to_matlab
export_model_to_ansys

print_time
```

Uncertainty in geometry

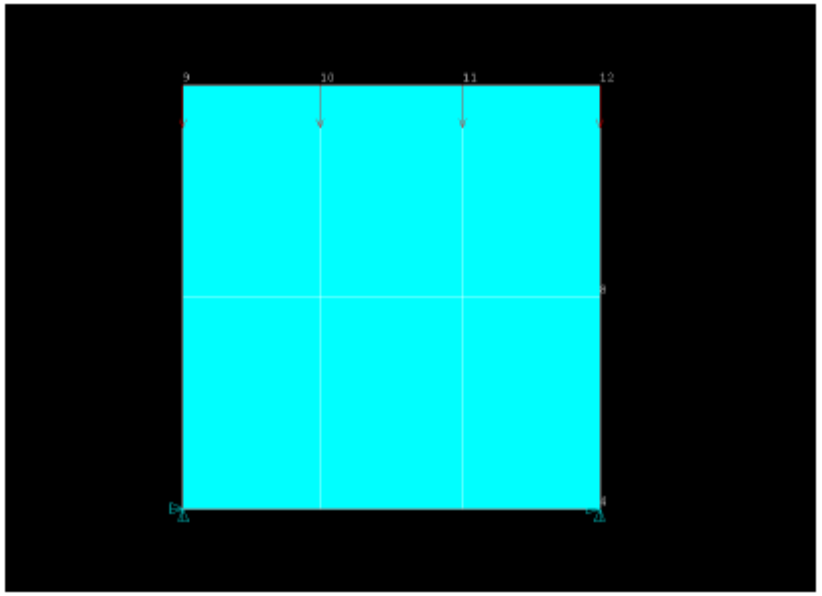


Figure 2: FEM model in ANSYS

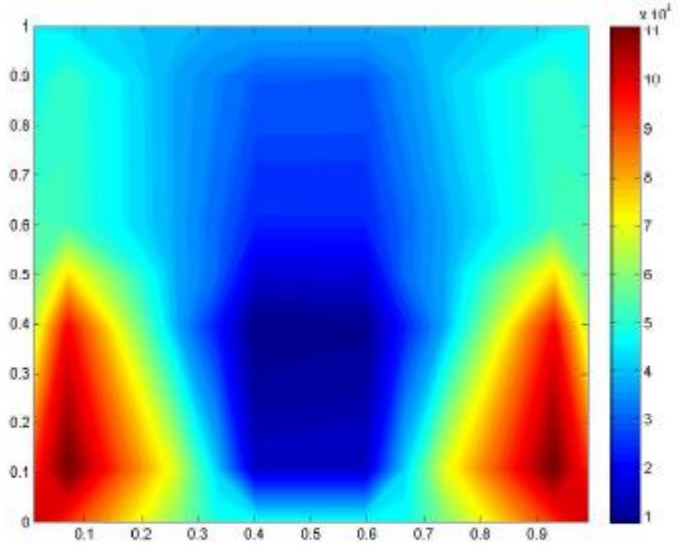


Figure 3: Interval von Mises stress

Andrzej Pownuk, Behzad Djafari-Rouhani, Naveen Kumar Goud Ramunigari,
Finite Element Method with the Interval Set Parameters
and its Applications in Computational Science
AMERICAN CONFERENCE ON APPLIED MATHEMATICS (AMERICAN-MATH '10)
University of Harvard, Cambridge, USA, January 27-29, 2010.
ISBN: 978-960-474-150-2, ISSN: 1790-2769, pp. 310-315.

Uncertainty in geometry

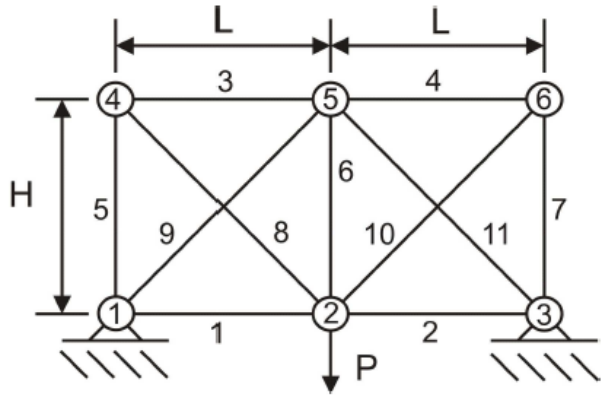


Figure 5: 11 bar truss

Table 1: Interval displacement of the truss with uncertain geometry

	Lower bound $[m]$	Upper bound $[m]$
u_{2x}	-2.883822e-005	2.886680e-005
u_{2y}	-1.526831e-002	-1.463698e-002
u_{5x}	-7.216225e-005	7.214392e-005
u_{5y}	-1.296921e-002	-1.239865e-002

Time dependent solution

$$\ddot{y} + \omega^2 y = 0$$

$$A \in \left[\underline{A}, \overline{A}\right] = [1,2], \quad B \in \left[\underline{B}, \overline{B}\right] = [1,2]$$

$$y = A\cos\omega t + B\sin\omega t$$

$$\underline{y}(t) = \begin{cases} 1\cos(t) + 1\sin(t), t \in \left[0, \dfrac{\pi}{2}\right] \\ 2\cos(t) + 1\sin(t), t \in \left[\dfrac{\pi}{2}, \pi\right] \\ 2\cos(t) + 2\sin(t), t \in \left[\pi, \dfrac{3\pi}{2}\right] \\ 1\cos(t) + 2\sin(t), t \in \left[\dfrac{3\pi}{2}, 2\pi\right] \end{cases},$$

Time dependent solutions

Interval equation y'=p*cos(p*t)

p1= 1 p2= 1.2

Number of timesteps = 200

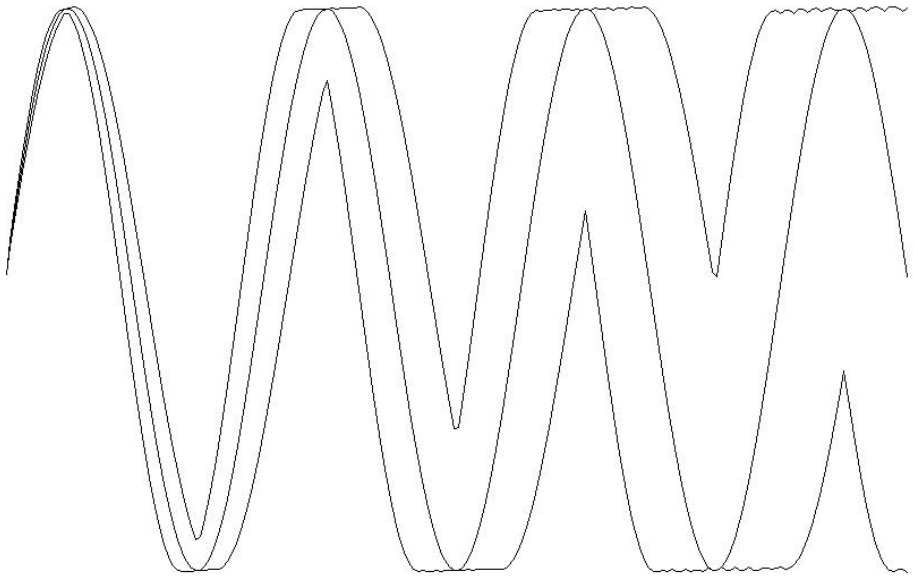
dt = 0.1

Initial value

u0= 1

Number of interval parameters 1

Calculate | Print u | Print dudp



u(p0)	du(p0)/dp	min u(p)	max u(p)	pMin	pMax	Sign
1	0	1	1	1.1	1.1	0
1.10933517077524	0.0981880484864608	1.0995004165278	1.11913703630246	1	1.2	0.0981880484
1.2166838902016	0.190976741711742	1.19750707431193	1.23569759328471	1	1.2	0.1909767417
1.32074854798973	0.27488755612755	1.29304072322449	1.34800521212606	1	1.2	0.2748875561
1.42027123094392	0.346621385986643	1.38514682262478	1.45444460285957	1	1.2	0.3466213859
1.51404892837047	0.403126040601407	1.47290507881381	1.55348487664874	1	1.2	0.4031260406

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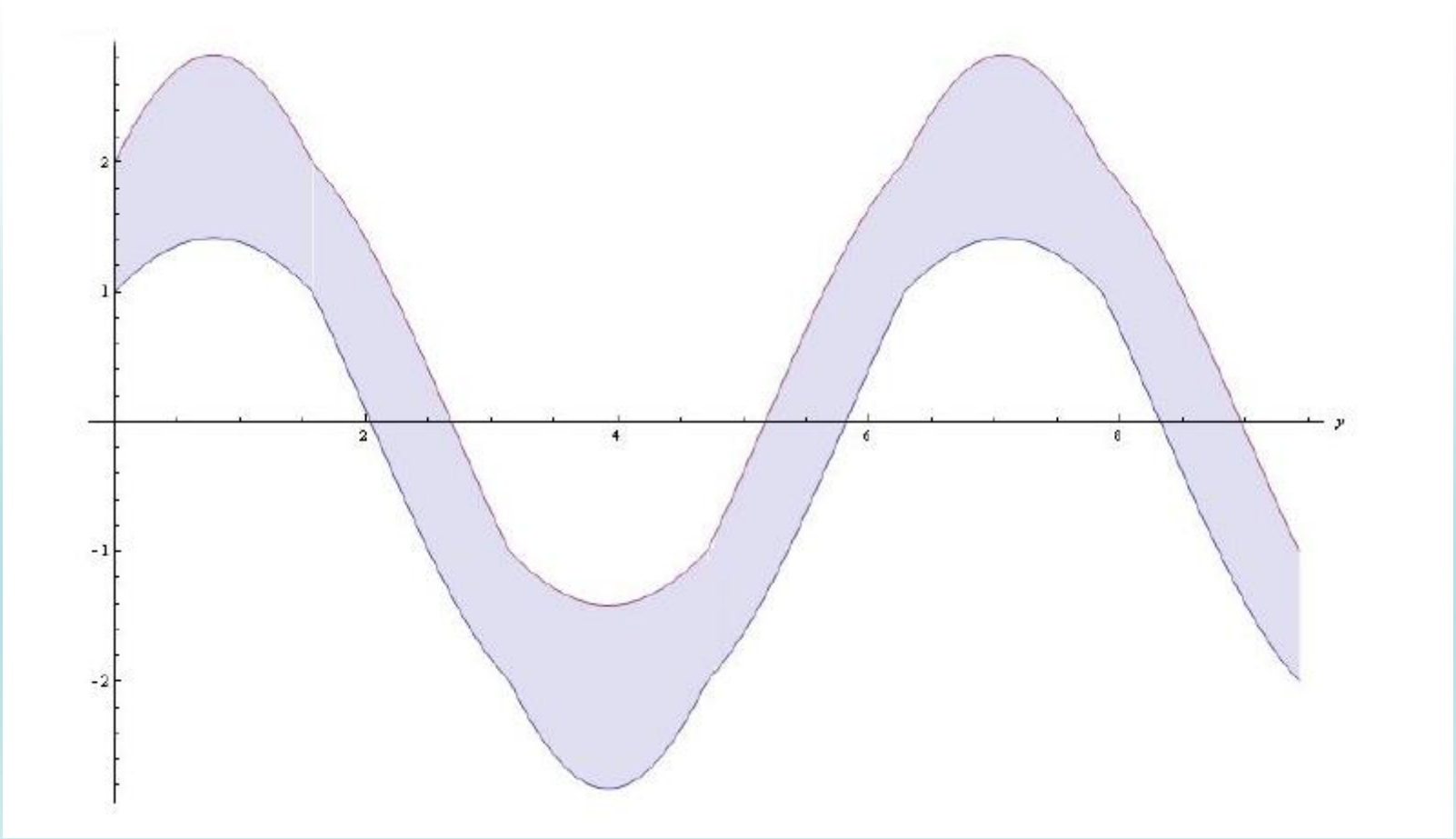
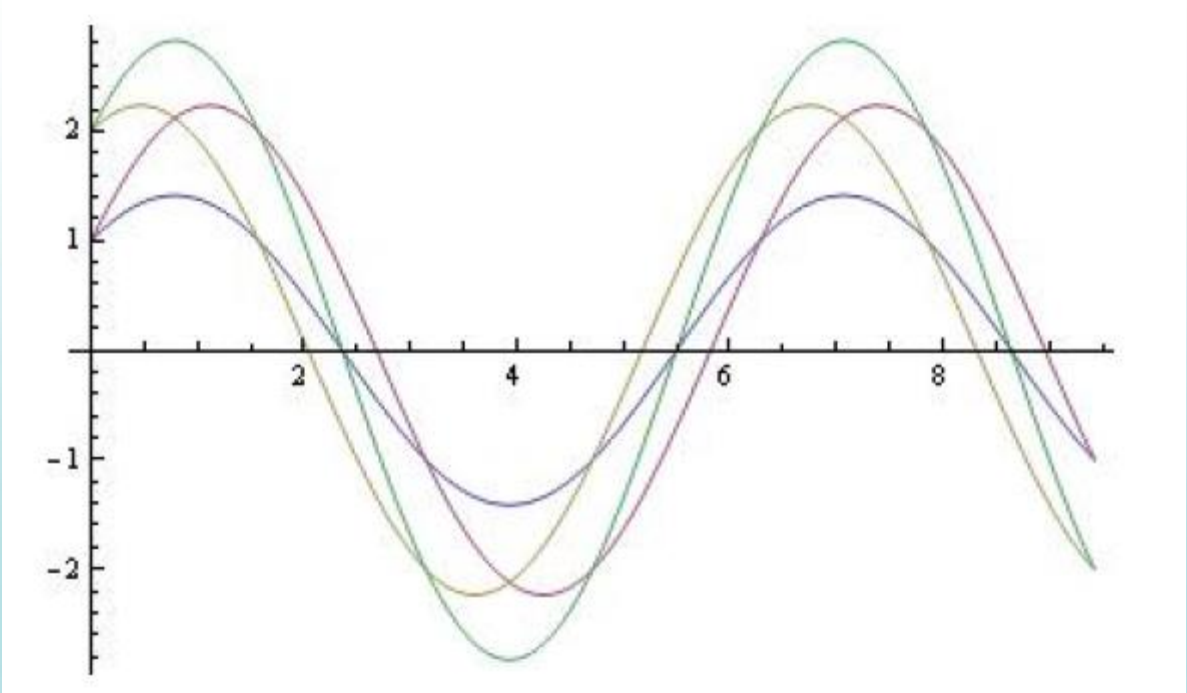
Interval solution



Combinatoric solution

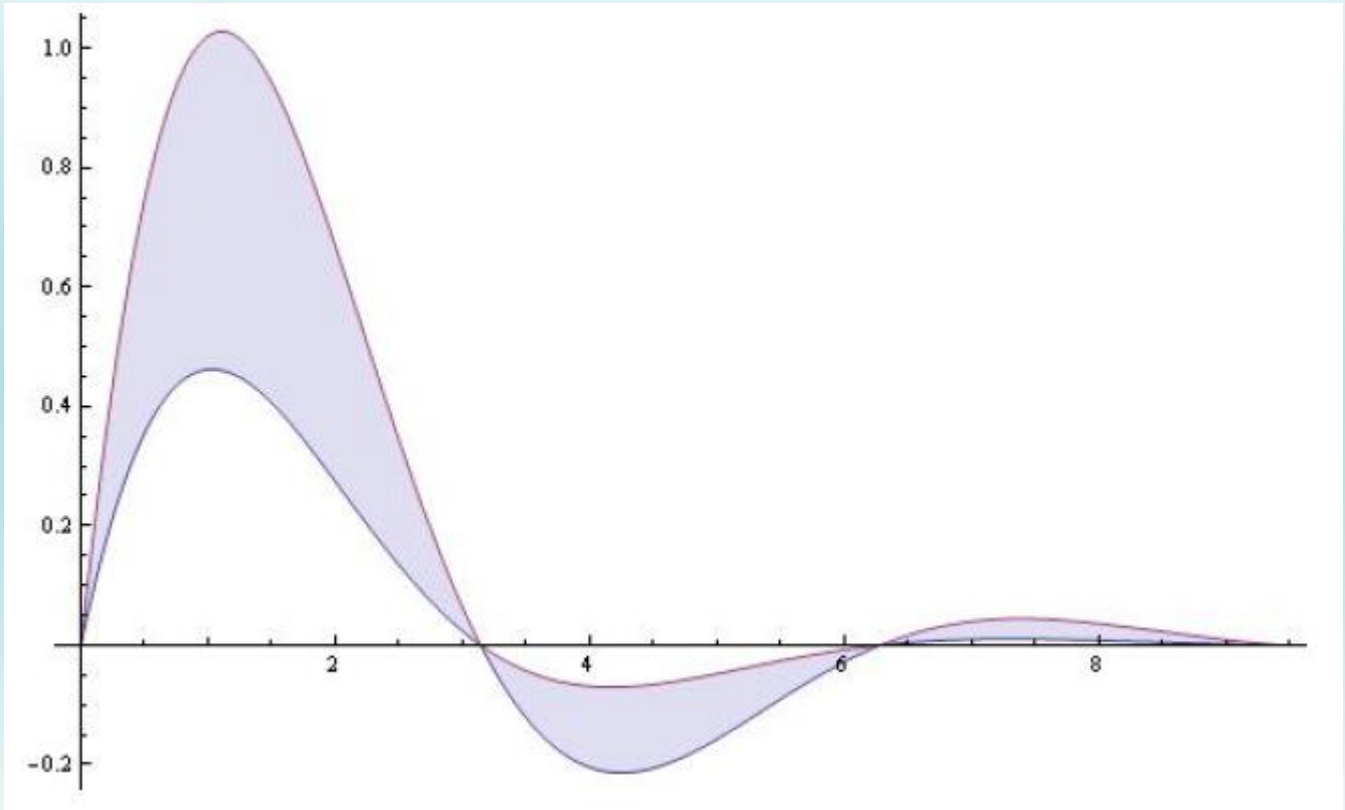


Damped vibrations

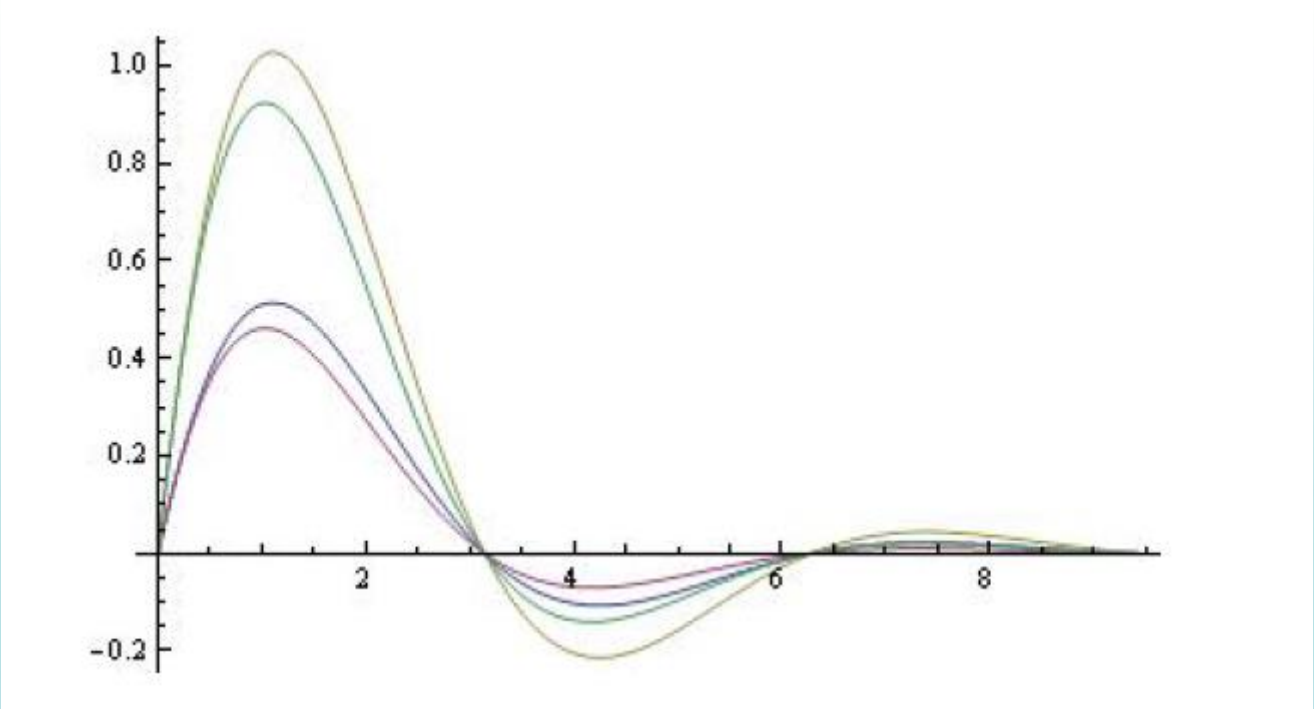
$$\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = 0$$

$$A \in \left[\underline{A}, \overline{A}\right] = [1, 2], \ \beta \in [0.5, 0.6], \ \varphi = 0, \ \omega = 1$$

$$y = Ae^{-\beta t}\sin(\omega t + \varphi)$$



Combinatoric solution



Numerical integration

$$\begin{bmatrix} \dot{y} \\ \dot{\mathrm{v}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} y \\ \mathrm{v} \end{bmatrix}$$

$$\dot{x} = Ax$$

$$\dot{u}^{(i)} = \frac{\partial A}{\partial p_i} x + A u^{(i)}$$

$$\dot{u}^{(i)} = \frac{\partial \dot{x}}{\partial p_i} = \frac{\partial}{\partial p_i} \begin{bmatrix} y \\ \mathrm{v} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial y}{\partial p_i} \\ \dfrac{\partial \mathrm{v}}{\partial p_i} \end{bmatrix}$$

$$x_{k+1} = x_k + A x_k \Delta t$$

$$u_{k+1}^{(i)} = u_k^{(i)} + \left(\frac{\partial A}{\partial p_i} x_k + A u_k^{(i)} \right) \Delta t$$

Numerical integration

$$\begin{bmatrix} \left(\frac{dy}{d\,\mathrm{v}_0}\right)_{k+1} \\ \left(\frac{d\,\mathrm{v}}{d\,\mathrm{v}_0}\right)_{k+1} \end{bmatrix} = \begin{bmatrix} \left(\frac{dy}{d\,\mathrm{v}_0}\right)_{k} \\ \left(\frac{d\,\mathrm{v}}{d\,\mathrm{v}_0}\right)_{k} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -\omega_k^2 & 0 \end{bmatrix} \begin{bmatrix} \left(\frac{dy}{d\,\mathrm{v}_0}\right)_{k} \\ \left(\frac{d\,\mathrm{v}}{d\,\mathrm{v}_0}\right)_{k} \end{bmatrix} \Delta t$$

$$\text{If } \left(\frac{dy}{dy_0}\right)_k \geq 0, \text{ then } p_{1,k}^{\min,y} = \underline{p}_1,\ p_{1,k}^{\max,y} = \overline{p}_1.$$

$$\text{If } \left(\frac{dy}{dy_0}\right)_k < 0, \text{ then } p_{1,k}^{\min,y} = \overline{p}_1,\ p_{1,k}^{\max,y} = \underline{p}_1.$$

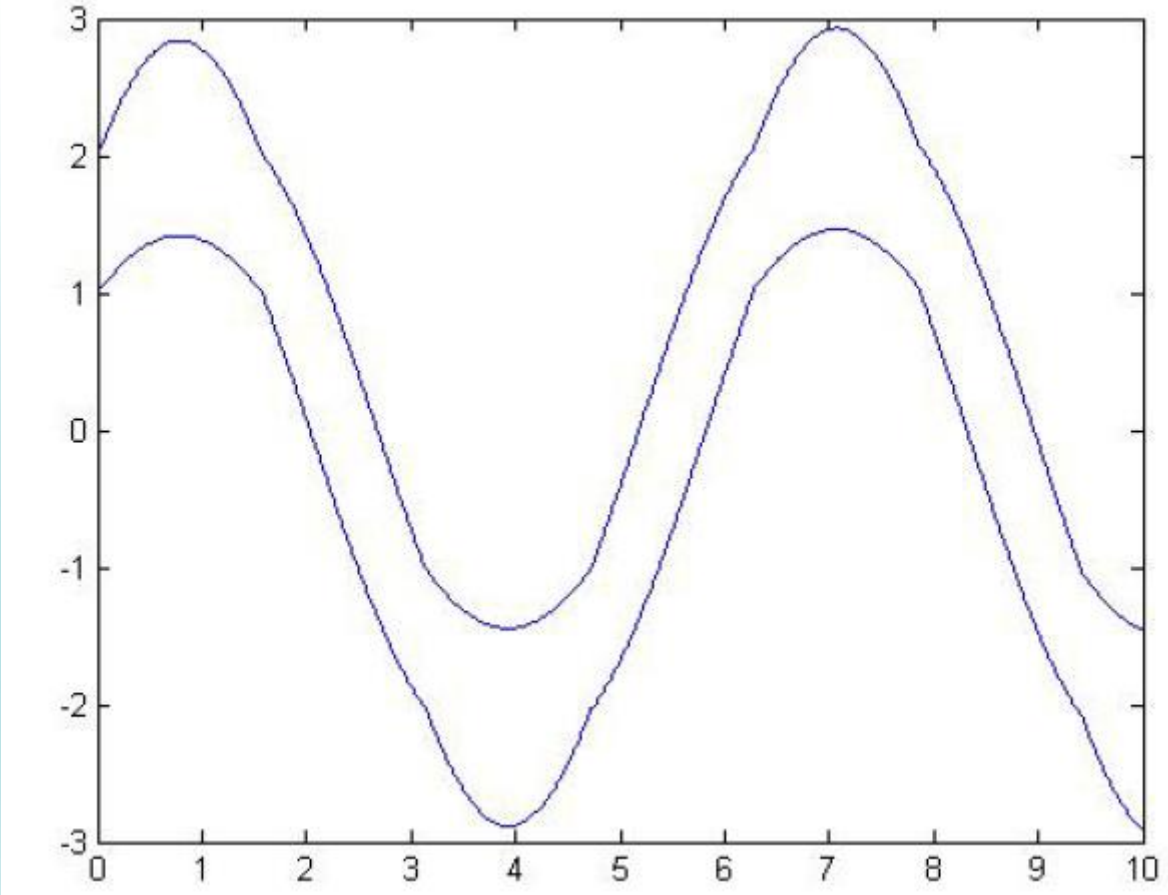
Interval solution

$$\underline{y}_{k+1} = y_{k+1}\left(p_{1,k}^{\min,y}, p_{2,k}^{\min,y}\right)$$

$$\overline{y}_{k+1} = y_{k+1}\left(p_{1,k}^{\max,y}, p_{2,k}^{\max,y}\right)$$

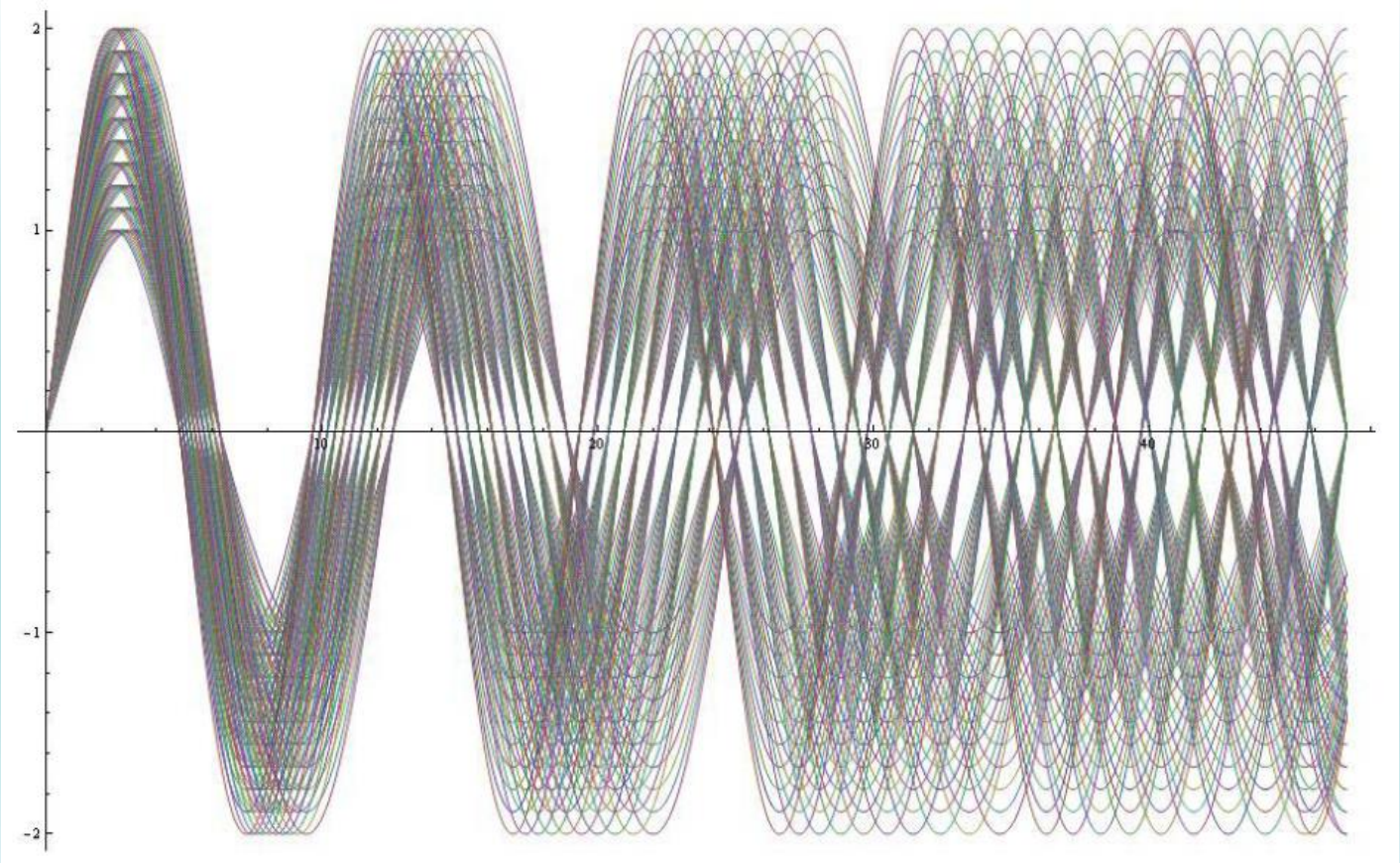
$$\underline{\mathrm{v}}_{k+1} = \mathrm{v}_{k+1}\left(p_{1,k}^{\min,\mathrm{v}}, p_{2,k}^{\min,\mathrm{v}}\right)$$

$$\overline{\mathrm{v}}_{k+1} = \mathrm{v}_{k+1}\left(p_{1,k}^{\max,\mathrm{v}}, p_{2,k}^{\max,\mathrm{v}}\right)$$



Nonmonotone example

$$y = A \sin \omega t \qquad A \in [1,2],\ \omega \in [0.5, 0.65]$$



Response surface method (Hermitte interpolation)

Interval equation y'=p*cos(p*t)

p1= 1 p2= 1.05

Number of timesteps = 200

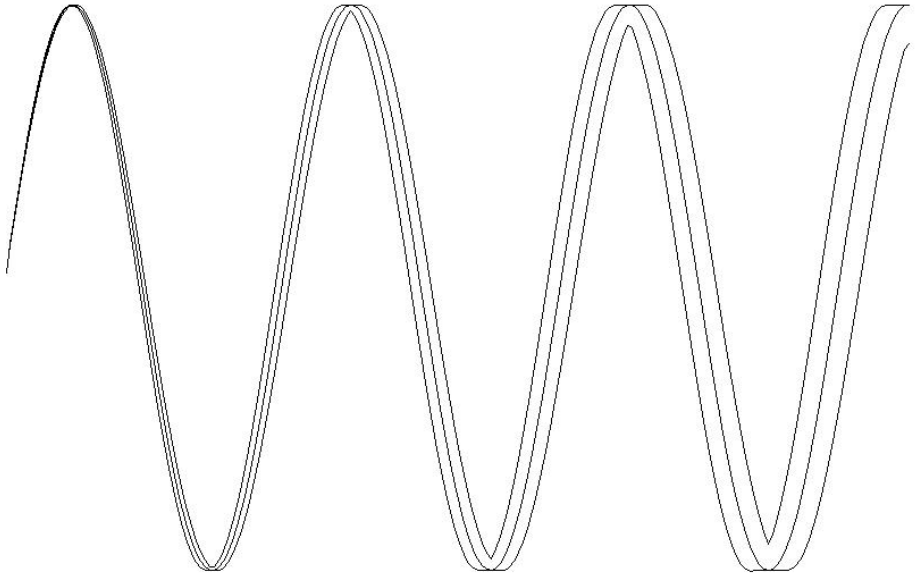
dt = 0.1

Initial value

u0= 1

Number of interval parameters 1

Calculate | Print u | Print dudp



u(p0)	du(p0)/dp	min u(p)	max u(p)	pMin	pMax	Sign
1	0	1	1	1.025	1.025	0
1.10196202594252	0.098426360982981	1.0995004165278	1.10442171908859	1	1.05	0.0984263609
1.20231577685577	0.192159332656884	1.19750707431193	1.20711496513463	1	1.05	0.1921593326
1.30000783391903	0.278161343431539	1.29304072322449	1.3069485849206	1	1.05	0.2781613434
1.39401271825773	0.353530443265838	1.38514682262478	1.4028229236534	1	1.05	0.3535304432
1.48334365545139	0.41555175112159	1.47290507881381	1.4936819375238	1	1.05	0.4155517511

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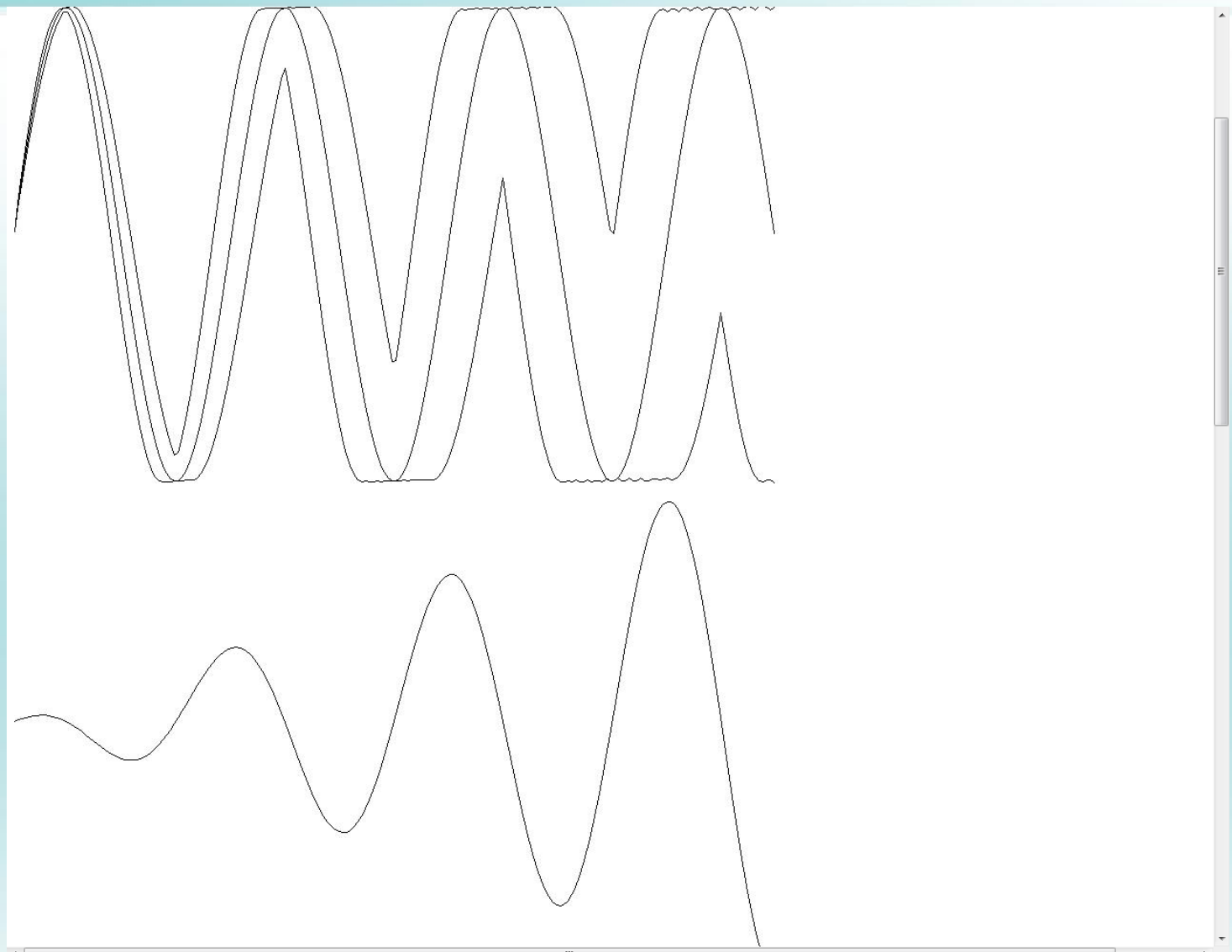
Time dependent solution



Numerical values

u(p0)	du(p0)/dp	min u(p)	max u(p)	pMin	pMax	Sign
1	0	1	1	1.1	1.1	0
1.10933517077524	0.0981880484864608	1.0995004165278	1.11913703630246	1	1.2	0.0981880484
1.2166838902016	0.190976741711742	1.19750707431193	1.23569759328471	1	1.2	0.1909767417
1.32074854798973	0.27488755612755	1.29304072322449	1.34800521212606	1	1.2	0.2748875561
1.42027123094392	0.346621385986643	1.38514682262478	1.45444460285957	1	1.2	0.3466213859
1.51404892837047	0.403126040601407	1.47290507881381	1.55348487664874	1	1.2	0.4031260406
1.60094807383518	0.441659551520897	1.55543864030478	1.64370156414564	1	1.2	0.4416595515
1.67991824749238	0.459848205107685	1.63192285903323	1.7237971032466	1	1.2	0.4598482051
1.75000487335423	0.455738298184433	1.70159352996795	1.79261950157529	1	1.2	0.4557382981
1.8103607580182	0.427840712361141	1.76375452679501	1.84917890527614	1	1.2	0.4278407123
1.86025633137501	0.37516751489694	1.81778475738183	1.89266183581334	1	1.2	0.3751675148
1.89908846550914	0.297259918830914	1.86314436952438	1.92244289001163	1	1.2	0.2972599188
1.9263877651909	0.19420707078054	1.89938014497205	1.93809373506021	1	1.2	0.1942070707
1.94182424183404	0.066655279236997	1.92613003783449	1.94303598488951	1	1.14	False
1.94521130233311	-0.0841925527680002	1.92631087263913	1.94689702338667	1.2	1.06	False
1.93650800456437	-0.25658737134357	1.89904660747524	1.95020046229131	1.2	1	-0.256587371
1.91581955228616	-0.448254215049079	1.85798864933714	1.94728051006118	1.2	1	-0.448254215
1.88339602345646	-0.656422024000306	1.80372750988023	1.93439606063162	1.2	1	-0.656422024
1.83962934733957	-0.877862626525096	1.73704361233015	1.91167585116232	1.2	1	-0.877862626
1.78504856694191	-1.10893836573583	1.65889605306685	1.87934689447596	1.2	1	-1.108938365
1.72031344404382	-1.34565768630167	1.5704088072019	1.83773221082125	1.2	1	-1.345657686
1.64620648412842	-1.58373786909287	1.47285456274991	1.78724760036127	1.2	1	-1.583737869
1.56362347760861	-1.81867398027498	1.3676364159026	1.72839748863573	1.2	1	-1.818673980
1.4735626716881	-2.04581299373643	1.25626769067748	1.66176988650775	1.2	1	-2.045812993
1.37711270374473	-2.26043195313286	1.14035017318972	1.58803051495362	1.2	1	-2.260431953
1.27543944209516	-2.4578189638105	1.02155107359767	1.50791615339893	1.2	1	-2.457818963
1.16977189320705	-2.63335574667414	0.901579047072116	1.42222727806204	1.2	1	-2.633355746
1.06138734570969	-2.78260044668841	0.782159618673256	1.33182006386033	1.2	1	-2.782600446
0.951595930779219	-2.90136936886634	0.665010365592567	1.23759782979346	1.2	1	-2.901369368
0.841724785529725	-2.98581631474954	0.55181621370675	1.1405020132785	1.2	1	-2.985816314
0.73310201083975	-3.03250821267315	0.444205203746652	1.04150276361846	1.2	1	-3.032508212
0.627040617152909	-3.03849577539512	0.343725075629787	0.941589248591132	1.2	1	-3.038495775
0.524822654941236	-3.00137797852187	0.25182100773763	0.841759771011656	1.2	1	-3.001377978
0.427683713782084	-2.91935923186057	0.16981483130763	0.74301179402077	1.2	1	-2.919359231
0.336797990541544	-2.79129821237115	0.0988860188936661	0.646331974762824	1.2	1	-2.791298212
0.253264094036418	-2.61674744051482	0.0400547203327821	0.552686306033744	1.2	1	-2.616747440
0.178091765642251	-2.39598280998079	-0.00583290979273951	0.463010464400329	1.2	1	-2.395982809
0.112189673737405	-2.130022422276	-0.0381168821668274	0.378200461229289	1.2	1	-2.130022422
0.0563544298977249	-1.82063423052763	-0.05633286518663	0.299103690037847	1.2	1	-1.820634230
0.0112609596118714	-1.47033215894515	-0.0602188648303894	0.226510459373962	1.2	1	False
-0.0225456560857547	-1.08236053343161	-0.0584581856751809	0.161146097204544	1.16	1	False
-0.0446567694477693	-0.660666832421701	-0.0573692217783774	0.103663702569416	1.14	1	False
-0.0548051056677155	-0.209862942666894	-0.0553893750737604	0.0546376204053498	1.12	1	False
-0.0528679936455617	0.264824720153285	-0.0543829395818118	0.0655671726390086	1.08	1.2	False
-0.0388688488119254	0.757615693995072	-0.0534588783030578	0.130081480352221	1.06	1.2	False
-0.0129768900869358	1.26224585764442	-0.052260177477077	0.206244626198416	1.04	1.2	False
0.0244949056049213	1.7720468655001	-0.0509660321464551	0.29296117642695	1.02	1.2	False
0.073093586538549	2.28003362484394	-0.0497092824446369	0.388983907251351	1	1.2	2.2800336248
0.132231701379954	2.77899867512722	-0.0409593841006922	0.492931754926759	1	1.2	2.7789986751
0.201194400182883	3.26161220822367	-0.0223081471584346	0.603309663699155	1	1.2	3.2616122082
0.279148075354921	3.72052636471651	0.00605807138788801	0.718530098097199	1	1.2	3.7205263647
0.36515043814276	4.14848235578861	0.0438558456591861	0.836935873919226	1	1.2	4.1484823557
0.458161908834033	4.53841889461124	0.0907075127892238	0.956823993063353	1	1.2	4.5384188946
0.557058182993217	4.88358037645473	0.14614494640714	1.07647013728646	1	1.2	4.8835803764
0.660643821833382	5.17762322396105	0.209614234001403	1.19415346860497	1	1.2	5.1776232239
0.76766670244604	5.41471881371187	0.280481211430529	1.30818137963999	1	1.2	5.4147188137
0.876833153217563	5.58965142267162	0.358037799281554	1.41691383792938	1	1.2	5.5896514226
0.986823591478194	5.69790967824	0.44150907776547	1.51878697406901	1	1.2	5.6979096782
1.09630847435884	5.73577006314883	0.530061029459602	1.6123355744215	1	1.2	5.7357700631
1.20396437004479	5.70037111560069	0.622808872534006	1.69621415488586	1	1.2	5.7003711156

Big uncertainty

Interval equation y'=p*cos(p*t)

p1= 1 p2= 1.5

Number of timesteps = 200

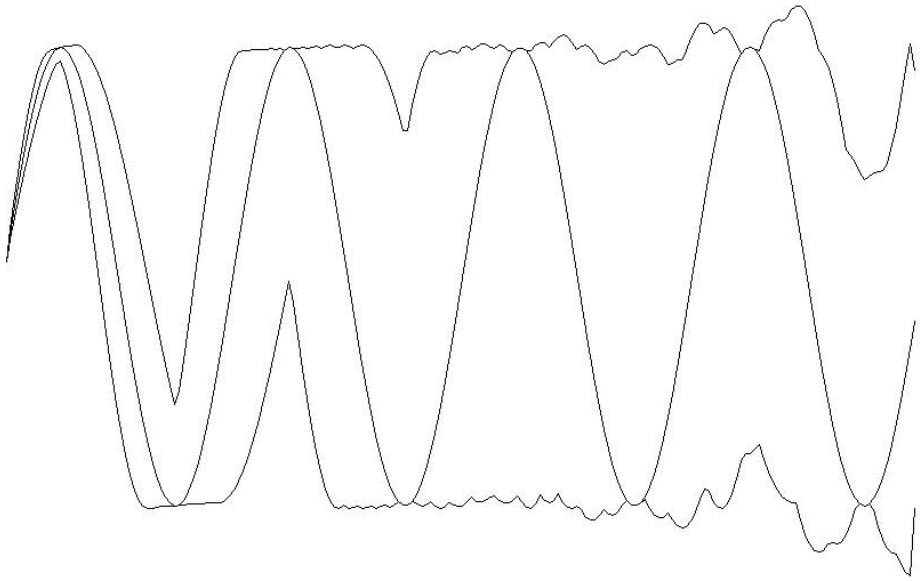
dt = 0.1

Initial value

u0= 1

Number of interval parameters 1

Calculate | Print u | Print dudp



u(p0)	du(p0)/dp	min u(p)	max u(p)	pMin	pMax	Sign
1	0	1	1	1.25	1.25	0
1.12402470840367	0.0976613325556176	1.0995004165278	1.14831566169041	1	1.5	0.0976613325
1.2451387611175	0.188367475745319	1.19750707431193	1.29161613505925	1	1.5	0.1883674757
1.36145221385654	0.267683018095824	1.29304072322449	1.42668320041215	1	1.5	0.2676830180
1.47115003409283	0.331469997354651	1.38514682262478	1.5504835426486	1	1.5	0.3314699973
1.57252042403098	0.375997729746394	1.47290507881381	1.66023687297967	1	1.5	0.3759977297

Done Internet | Protected Mode: On 100%

Big uncertainty – numerical values

u(p0)	du(p0)/dp	min u(p)	max u(p)	pMin	pMax	Sign
1	0	1	1	1.25	1.25	0
1.12402470840367	0.0976613325556176	1.0995004165278	1.14831566169041	1	1.5	0.0976613325
1.2451387611175	0.188367475745319	1.19750707431193	1.29161613505925	1	1.5	0.1883674757
1.36145221385654	0.267683018095824	1.29304073222449	1.42668320041215	1	1.5	0.2676830180
1.47115003409283	0.331469997354651	1.38514682262478	1.5504835426486	1	1.5	0.3314699973
1.57252042403098	0.375997729746394	1.47290507881381	1.66023687297967	1	1.5	0.3759977297
1.66398153264021	0.398043709632026	1.55543864030478	1.75347836822027	1	1.5	0.3980437096
1.74410613991063	0.394983339002706	1.63192285903323	1.82811402540403	1	1.5	0.3949833390
1.81164392814415	0.36486647110873	1.70159352996795	1.88246768857553	1	1.5	0.3648664711
1.86554099274398	0.306479018452449	1.76375452679501	1.91531869163949	1	1.5	0.3064790184
1.90495628804339	0.219388177272527	1.81778475738007	1.92720955230245	1	1.45	False
1.92927475154201	0.103970152730742	1.86314436952301	1.93309236541129	1	1.35	False
1.93811690175047	-0.038580375093096	1.87998082435845	1.93811690174682	1.5	1.25	False
1.93134475987211	-0.206259431433512	1.82445369965255	1.94288401724882	1.5	1.15	False
1.90906400291592	-0.396281577014	1.74872678398758	1.94679319970942	1.5	1.05	False
1.87162231464222	-0.605126012372175	1.65450074056359	1.95020046229131	1.5	1	-0.605126012
1.81960396007383	-0.828600181392026	1.5438916832324	1.94728051006118	1.5	1	-0.828600181
1.75382066823579	-1.06191977019888	1.41938365294712	1.93439606063162	1.5	1	-1.061919770
1.67529896539545	-1.29980360177093	1.28377283164456	1.91167585116232	1.5	1	-1.299803601
1.58526415646336	-1.53658164400551	1.1401047460116	1.87934689447596	1.5	1	-1.536581644
1.48512120452	-1.76631404158619	0.991605871521529	1.8377322082125	1.5	1	-1.766314041
1.37643280683675	-1.98291883811793	0.841611172750828	1.78724760036127	1.5	1	-1.982918838
1.26089500950769	-2.18030584879557	0.693489207264498	1.72839748863573	1.5	1	-2.180305848
1.14031074121753	-2.35251398651969	0.550566475075352	1.66176988650775	1.5	1	-2.352513986
1.01656167914247	-2.49384923859769	0.41605271262523	1.58803051495362	1.5	1	-2.493849238
0.89157888600726	-2.59902043942753	0.292968809024296	1.50791615339893	1.5	1	-2.599020439
0.767312676497192	-2.6632699883133	0.184078963394275	1.42222727806204	1.5	1	-2.663269988
0.645702183249761	-2.68249672122556	0.0918286069154391	1.33182006386033	1.5	1	-2.682496721
0.528645097338411	-2.65336826026328	0.0182894837143342	1.23759782979346	1.5	1	-2.653368260
0.417968055438034	-2.57342033312212	-0.0348868760428855	1.1405020132785	1.5	1	-2.573420333
0.315398135770589	-2.4411407743277	-0.0665062459575025	1.04150276361846	1.5	1	-2.441140774
0.222535907627089	-2.25603618667537	-0.0758585231485512	0.941589248591132	1.5	1	-2.256036186
0.140830455019137	-2.01867955063856	-0.0734039206363857	0.841759771013088	1.45	1	False
0.0715567642075853	-1.73073741603264	-0.0706280802306196	0.74301179401909	1.4	1	False
0.0157958279683612	-1.39497568777687	-0.0668115369767825	0.646331974761843	1.35	1	False
-0.0255822229382497	-1.01524341937048	-0.0651226845586628	0.55268630603274	1.35	1	False
-0.0519316978670972	-0.596434446364269	-0.065200898034476	0.463010464399332	1.3	1	False
-0.0628414220747266	-0.144427119955376	-0.0628414220744844	0.37820046122944	1.25	1	False
-0.0581411529637295	0.333997170096727	-0.0581411529622073	0.437950839068222	1.25	1.5	False
-0.0379042366615374	0.83125557179927	-0.0602638247586071	0.574095830930872	1.2	1.5	False
-0.00244646347863412	1.33908392767716	-0.0567002276343089	0.718121373927346	1.15	1.5	False
0.0476788598936205	1.84867360840092	-0.0564161802653018	0.866792964869205	1.15	1.5	False
0.111689544548851	2.35082276517406	-0.0550517245059865	1.01677176032103	1.1	1.5	False
0.188586725161939	2.83609942534465	-0.0531295779860557	1.16468955989092	1.1	1.5	False
0.277170446948347	3.29501358183749	-0.0523197360327572	1.30722444870008	1.05	1.5	False
0.376058390557189	3.71819520234913	-0.0509971135246587	1.44117540038667	1.05	1.5	False
0.483707442702379	4.09657491362735	-0.0484704161024183	1.56353416539355	1	1.5	False
0.598437775929349	4.42156400345496	-0.0497092824446369	1.67155282966768	1	1.5	4.4215640034
0.718459061760645	4.68523033103935	-0.0409593841006922	1.76280552684752	1	1.5	4.6852303310
0.841898408173027	4.88046674716175	-0.0223081471584346	1.83524291809591	1	1.5	4.8804667471
0.96682958545109	5.00114869932642	0.00605807138788801	1.88723821577116	1	1.5	5.0011486993
1.09130308435772	5.04227783378924	0.0438558456591861	1.9176237175827	1	1.5	5.0422778337
1.21337653757372	5.00010860410496	0.0907075127910275	1.92571703066523	1	1.5	False
1.33114502969019	4.87225515195232	0.14614494640864	1.92404633285815	1	1.45	False
1.44277082277632	4.65777603661931	0.209614233998082	1.92763154143475	1	1.45	False
1.54651203366523	4.35723474976792	0.280481211431152	1.92823815838662	1	1.4	False
1.64074981545815	3.9727343560991	0.358037799281192	1.93042816698727	1	1.4	False
1.72401361908882	3.50792504159259	0.441509077764522	1.93014449207379	1	1.35	False
1.79500414074991	2.96798382163812	0.530061029455737	1.93358429124035	1	1.35	False
1.85261359709428	2.35956615351286	0.622808872530413	1.9291063150034	1	1.3	False

Small uncertainty

Interval equation y'=p*cos(p*t)

p1= 1 p2= 1.05

Number of timesteps = 200

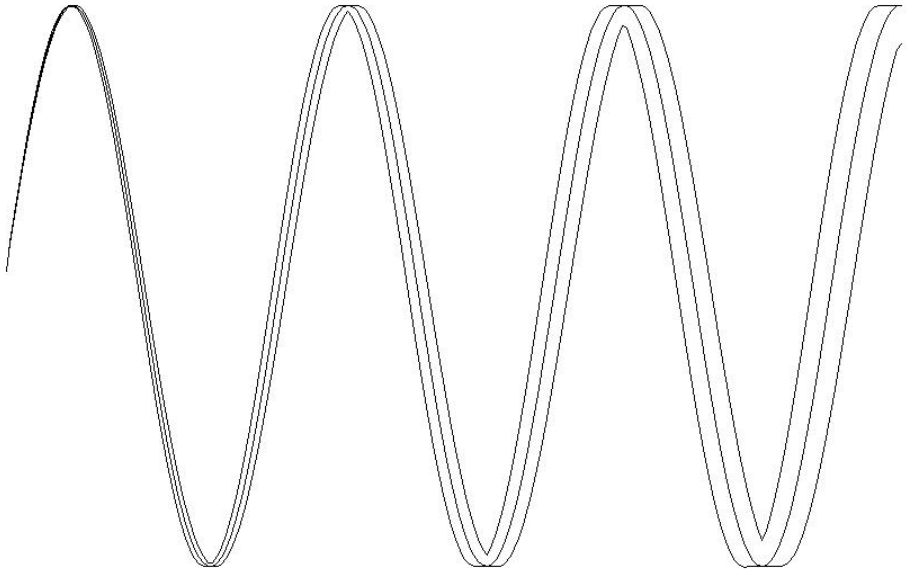
dt = 0.1

Initial value

u0= 1

Number of interval parameters 1

Calculate | Print u | Print dudp



u(p0)	du(p0)/dp	min u(p)	max u(p)	pMin	pMax	Sign
1	0	1	1	1.025	1.025	0
1.10196202594252	0.098426360982981	1.0995004165278	1.10442171908859	1	1.05	0.0984263609
1.20231577685577	0.192159332656884	1.19750707431193	1.20711496513463	1	1.05	0.1921593326
1.30000783391903	0.278161343431539	1.29304072322449	1.3069485849206	1	1.05	0.2781613434
1.39401271825773	0.353530443265838	1.38514682262478	1.4028229236534	1	1.05	0.3535304432
1.48334365545139	0.41555175112159	1.47290507881381	1.4936819375238	1	1.05	0.4155517511

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Small uncertainty – numerical values

u(p0)	du(p0)/dp	min u(p)	max u(p)	pMin	pMax	Sign
1	0	1	1	1.025	1.025	0
1.10196202594252	0.098426360982981	1.0995004165278	1.10442171908859	1	1.05	0.0984263609
1.20231577685577	0.192159332656884	1.19750707431193	1.20711496513463	1	1.05	0.1921593326
1.30000783391903	0.278161343431539	1.29304072322449	1.3069485849206	1	1.05	0.2781613434
1.39401271825773	0.353530443265838	1.38514682262478	1.4028229236534	1	1.05	0.3535304432
1.48334365545139	0.41555175112159	1.47290507881381	1.4936819375238	1	1.05	0.4155517511
1.56706293375872	0.461746146691121	1.55543864030478	1.57852482589658	1	1.05	0.4617461466
1.6442917473292	0.489915483306888	1.63192285903323	1.65641705500383	1	1.05	0.4899154833
1.71421942107618	0.498183645998044	1.70159352996795	1.72650065171716	1	1.05	0.4981836459
1.7761119203777	0.485032834926336	1.76375452679501	1.78800365401408	1	1.05	0.4850328349
1.82931955627812	0.449334519188579	1.81778475738183	1.84024861404271	1	1.05	0.4493345191
1.87328380530868	0.390374578381733	1.86314436952438	1.88266006012493	1	1.05	0.3903745783
1.90754317233894	0.307872228456585	1.89938014497205	1.91477083550465	1	1.05	0.3078722284
1.93173803491646	0.201992413216169	1.92613002783451	1.93622724402087	1	1.05	0.2019924132
1.94561441824337	0.0733514322495547	1.94312674212453	1.94679294602649	1	1.05	0.0733514322
1.94902666116373	-0.0769843310273263	1.9463515616399	1.95020046229131	1.05	1	-0.076984331
1.94193894517669	-0.247506622429077	1.93490795265472	1.94728051006118	1.05	1	-0.247506622
1.92442567042522	-0.436280416233451	1.91258816898777	1.93439606063162	1.05	1	-0.436280416
1.89667067471378	-0.640965749785999	1.87963806025509	1.91167585116232	1.05	1	-0.640965749
1.85896530375277	-0.858846048295588	1.83642056776922	1.87934689447596	1.05	1	-0.858846048
1.81170535288666	-1.0868626030031	1.78341172678623	1.83773221082125	1.05	1	-1.086862603
1.75538691240859	-1.32165477821925	1.72119542303711	1.78724760036127	1.05	1	-1.321654778
1.69060116007339	-1.55960544015405	1.65045696129968	1.72839748863573	1.05	1	-1.559605440
1.61802815547228	-1.79689102410232	1.57197551685252	1.66176988650775	1.05	1	-1.796891024
1.53842970141012	-2.02953558743463	1.48661555295703	1.58803051495362	1.05	1	-2.029535587
1.45264134721959	-2.25346813491086	1.3953172989031	1.50791615339893	1.05	1	-2.253468134
1.36156361795413	-2.46458245092407	1.29908639350148	1.42222727806204	1.05	1	-2.464582450
1.26615256152707	-2.65879863112569	1.19898280809848	1.33182006386033	1.05	1	-2.658798631
1.16740971302434	-2.83212547408662	1.09610917112461	1.23759782979346	1.05	1	-2.832125474
1.06637158153582	-2.98072287269388	0.991598622780802	1.1405020132785	1.05	1	-2.980722872
0.964098769862599	-3.1009633352144	0.886602333641311	1.04150276361846	1.05	1	-3.100963335
0.861664841311327	-3.18949176758002	0.782276824654287	0.941589248591132	1.05	1	-3.189491767
0.760145050441336	-3.2432826615248	0.679771228208684	0.841759771011656	1.05	1	-3.243282661
0.660605056058643	-3.25969385765202	0.580214630585206	0.74301179402077	1.05	1	-3.259693857
0.564089734937087	-3.23651608809842	0.484703635212622	0.646331974762824	1.05	1	-3.236516088
0.471612213689589	-3.17201754982635	0.394290283718618	0.552686306033744	1.05	1	-3.172017549
0.384143233922605	-3.06498281620214	0.309970467823296	0.463010464400329	1.05	1	-3.064982816
0.3026009623083	-2.9147454607713	0.23267295971686	0.378200461229289	1.05	1	-2.914745460
0.227841352538696	-2.72121384224873	0.163249181750498	0.299103690037847	1.05	1	-2.721213842
0.160649160332877	-2.48488958282349	0.102463828126033	0.226510459617833	1.05	1	-2.484889582
0.101729705813217	-2.2068783619404	0.0509864418852592	0.161146097531472	1.05	1	-2.206878361
0.0517014697214172	-1.88889274367975	0.00938403997736224	0.103663702878145	1.05	1	-1.888892743
0.0110896011922796	-1.53324685654393	-0.0218851323616192	0.054637620744075	1.05	1	-1.533246856
-0.0196795947655263	-1.14284284864003	-0.0424766491225585	0.0145577035360774	1.05	1	-1.142842848
-0.0402831322164945	-0.721149147630122	-0.0521636973325071	-0.0161755834617645	1.05	1	-0.721149147
-0.0505047346993402	-0.272170662079803	-0.0522556242387413	-0.0372526267645927	1.04	1	False
-0.0502371054928614	0.199588832378393	-0.0511532149737466	-0.0385199754491286	1.015	1.05	False
-0.0394830539154993	0.689171773691284	-0.0497092824446369	-0.0153372870885259	1	1.05	0.6891717736
-0.0183554658357686	1.19121891067572	-0.0409593841006922	0.0184498259826949	1	1.05	1.1912189106
0.0129238812968883	1.70002786948208	-0.0223081471584346	0.0624703101277115	1	1.05	1.7000278694
0.054026646462137	2.20961755020583	0.00605807138788801	0.116239285238105	1	1.05	2.2096175502
0.104521371682428	2.71379763003175	0.0438558456591861	0.179164492800879	1	1.05	2.7137976300
0.163878011061022	3.2062423728005	0.0907075127892238	0.250552819551654	1	1.05	3.2062423728
0.231473494702805	3.68056787753181	0.14614494640714	0.329617932030291	1	1.05	3.6805678775
0.306598269113107	4.13041184116139	0.209614234001403	0.415488937945271	1	1.05	4.1304118411
0.388463745419608	4.54951486437442	0.280481211430529	0.507219978942995	1	1.05	4.5495148643
0.476210577232927	4.93180229464227	0.358037799281554	0.603800649118842	1	1.05	4.9318022946
0.568917681252746	5.27146557795268	0.44150907776547	0.704167124511409	1	1.05	5.2714655779
0.665611905929674	5.56304208068089	0.530061029459602	0.807213880989939	1	1.05	5.5630420806
0.765278246690366	5.80149234584656	0.622808872534006	0.911805871463878	1	1.05	5.8014923458

Adaptive Taylor series

Second order approximaiton

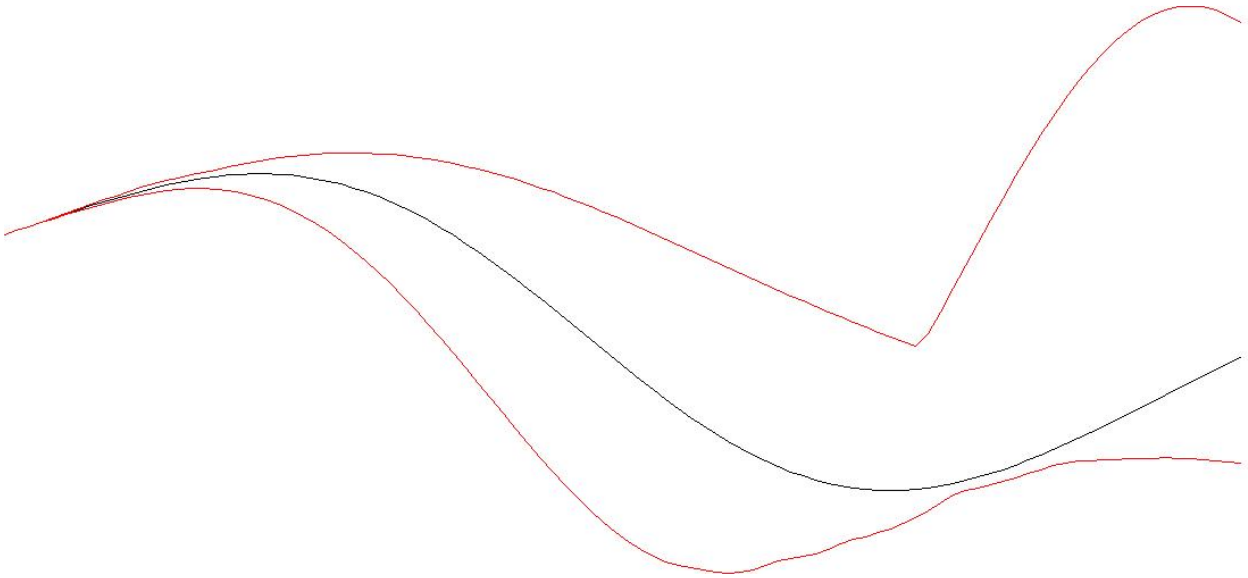
u''+(k/m)*u=(P/m)*cos(w*t)

v=u'

Calculation

u0= 1 v0= 1
P= 1
kMin= 1 kMax= 2
mMin= 1 mMax= 2
w= 2
number of intermediate points = 5

dt = 0.05 number of timesteps = 100



t	u	v	dudk	dvdk
0, 0	1	1	0	0
1, 0.05	1.05	0.999750208263901	0	-0.0333333333333333
2, 0.1	1.0999875104132	0.996253537155963	-0.00166666666666667	-0.0683333333333333

Done

Internet | Protected Mode: On

100%

Numerical values

t	u	v	dudk	dvdk
0, 0	1	1	0	0
1, 0.05	1.05	0.999750208263901	0	-0.0333333333333333
2, 0.1	1.0999875104132	0.996253537155963	-0.00166666666666667	-0.0683333333333333
3, 0.15	1.14980018727099	0.989020986091584	-0.00508333333333333	-0.104916250347107
4, 0.2	1.19925123657557	0.977584026428179	-0.0103291458506887	-0.142988756589473
5, 0.25	1.24813043789698	0.961500592693919	-0.0174785836801623	-0.182447340516124
6, 0.3	1.29620546753168	0.940360851544553	-0.0266009507059685	-0.223177759262016
7, 0.35	1.34322351010891	0.913792687532194	-0.0377598386690693	-0.26505456064444
8, 0.4	1.38891314448551	0.881466847494107	-0.0510125667012913	-0.307940685714616
9, 0.45	1.43298648686022	0.843101688683364	-0.0664096009870221	-0.351687162195736
10, 0.5	1.47514157129439	0.79846747963376	-0.0839939590968089	-0.396132898375059
11, 0.55	1.51506494527608	0.74739020714032	-0.103800604015562	-0.441104586130031
12, 0.6	1.55243445563309	0.68975484760035	-0.125855833322063	-0.486416720771789
13, 0.65	1.58692219801311	0.625508066249924	-0.150176669360653	-0.531871744293455
14, 0.7	1.61819760132561	0.554660313494281	-0.176770256575326	-0.57726031742586
15, 0.75	1.64593061700032	0.477287293511386	-0.205633272446619	-0.62236172464128
16, 0.8	1.66979498167589	0.393530786546305	-0.236751358678683	-0.666944141491896
17, 0.85	1.6894715210032	0.303598812747735	-0.270098579424631	-0.710766679707555
18, 0.9	1.70465146164059	0.20776513196292	-0.305636913410008	-0.753577468103097
19, 0.95	1.71503971823874	0.106368080537715	-0.343315786815163	-0.795117337820617
20, 1	1.72035812226562	-0.000191247201578662	-0.383071653706194	-0.835119539087817
21, 1.05	1.72034855990554	-0.111451458544853	-0.424827630660585	-0.873311227144694
22, 1.1	1.7147759869783	-0.226893942402897	-0.46849319201782	-0.909414797608517
23, 1.15	1.70343128985816	-0.345946542815803	-0.513963931898245	-0.943149337573569
24, 1.2	1.68613396271737	-0.467987793085774	-0.561121398776924	-0.974232183973929
25, 1.25	1.66273457306308	-0.592351671998989	-0.60983300797562	-1.00238057945899
26, 1.3	1.63311698946313	-0.71833283832059	-0.65995203694857	-1.02731341482898
27, 1.35	1.5972003475471	-0.845192294894599	-0.711317707690019	-1.04875304596366
28, 1.4	1.55494073280237	-0.972163429305387	-0.763755359988202	-1.06642717216406
29, 1.45	1.5063325613371	-1.09845837420299	-0.817076718596405	-1.08007076192473
30, 1.5	1.45140964262695	-1.22327462709986	-0.871080256692642	-1.08942801137281
31, 1.55	1.39024591127196	-1.34580186674487	-0.925551657261282	-1.09425431995908
32, 1.6	1.32295581793471	-1.46522890109821	-0.980264373259236	-1.09431826747175
33, 1.65	1.2496943728798	-1.58075068049039	-1.03498028663282	-1.08940357607328
34, 1.7	1.17065683885528	-1.69157530876335	-1.08945046543649	-1.07931104083763
35, 1.75	1.08607807341712	-1.79693098507066	-1.14341601747837	-1.06386041219431
36, 1.8	0.996231524163583	-1.89607280955822	-1.19660903808808	-1.04289221376763
37, 1.85	0.901427883685672	-1.98828938735192	-1.24875364877647	-1.01626947933535
38, 1.9	0.802013414318076	-2.07290916713192	-1.29956712274323	-0.983879393019379
39, 1.95	0.69836795596148	-2.14930645305783	-1.3487610923942	-0.945634817359487
40, 2	0.590902633308588	-2.21690703189909	-1.39604283326218	-0.90147569460516
41, 2.05	0.480057281713634	-2.27519336089118	-1.44111661799243	-0.85137030738567
42, 2.1	0.366297613669075	-2.3237092660439	-1.48368513336172	-0.795316385876503
43, 2.15	0.25011215036688	-2.36206410533135	-1.52345095265554	-0.733342049664053
44, 2.2	0.132008945100312	-2.38993635634862	-1.56011805513875	-0.665506573710172
45, 2.25	0.0125121272828816	-2.40707659357517	-1.59339338382425	-0.591900969123245
46, 2.3	-0.107841702395877	-2.41330982628607	-1.62298843228042	-0.512648370841462
47, 2.35	-0.22850719371018	-2.40853717433942	-1.64862085082249	-0.427904225814245
48, 2.4	-0.348934052427151	-2.39273686548194	-1.6700160621132	-0.337856276816114
49, 2.45	-0.468570895701248	-2.36596454438945	-1.68690887595401	-0.242724338629549
50, 2.5	-0.58686912292072	-2.32835289033123	-1.69904509288549	-0.142759864975141
51, 2.55	-0.703286767437282	-2.28011054704954	-1.70618308613424	-0.0382453062335089
52, 2.6	-0.817292294789759	-2.22152037511266	-1.70809535144592	0.07050674032111 25
53, 2.65	-0.928368313545392	-2.15293704356421	-1.70457001442986	0.1831545843864
54, 2.7	-1.0360151657236	-2.07478398408981	-1.69541228521054	0.299328695559407
55, 2.75	-1.13975436492809	-1.98754973708907	-1.68044585043257	0.418633148677387
56, 2.8	-1.23913185178255	-1.89178372491715	-1.6595141929987	0.540647253363285
57, 2.85	-1.3337210380284	-1.78809149308606	-1.63248183033054	0.664927358072639
58, 2.9	-1.42312561268271	-1.67712946533758	-1.59923546242691	0.791008817523446

Numerical values

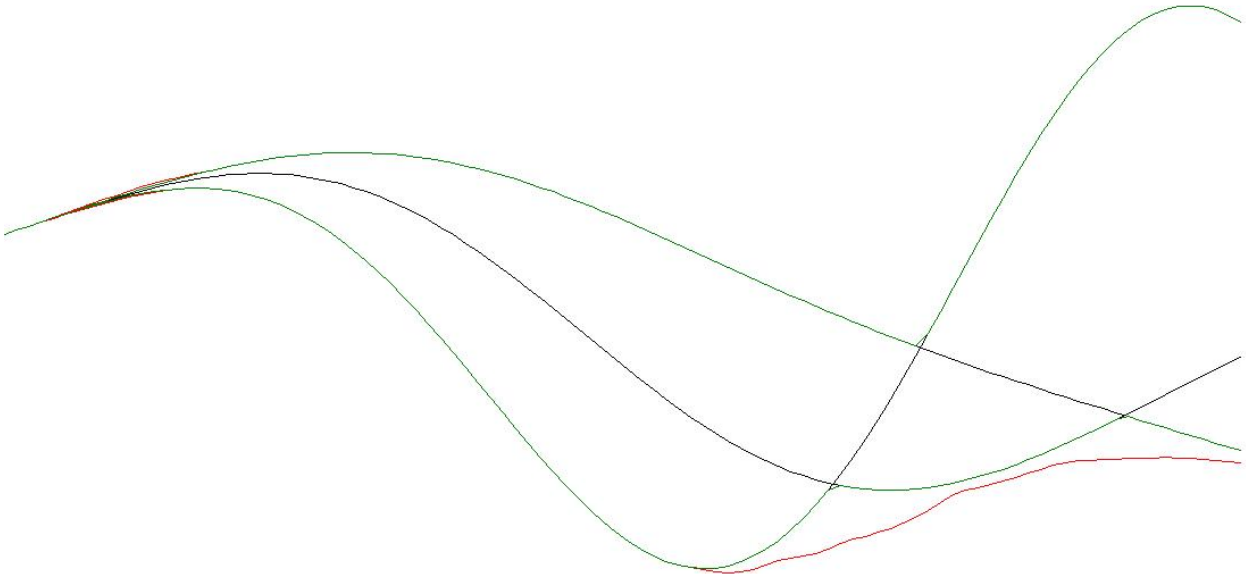
min u	max u	min v	max v	min u - combinations
1	1	1	1	0
1.05	1.05	0.999750208263901	0.999750208263901	0
1.0999875104132	1.0999875104132	0.996253537155963	0.996253537155963	0
1.14980018727099	1.14980018727099	0.989020986091584	0.989020986091584	0
1.19925123657557	1.19925123657557	0.977584026428179	0.977584026428179	0
1.24813043789698	1.24813043789698	0.961500592693919	0.961500592693919	0
1.29620546753168	1.29620546753168	0.940360851544553	0.940360851544553	0
1.34322351010891	1.34322351010891	0.913792687532194	0.913792687532194	0
1.38891314448551	1.38891314448551	0.881466847494107	0.881466847494107	0
1.43298648686022	1.43298648686022	0.843101688683364	0.843101688683364	0
1.47514157129439	1.47514157129439	0.79846747963376	0.79846747963376	0
1.51506494527608	1.51506494527608	0.74739020714032	0.74739020714032	0
1.55243445563309	1.55243445563309	0.68975484760035	0.68975484760035	0
1.58692219801311	1.58692219801311	0.625508066249924	0.625508066249924	0
1.61819760132561	1.61819760132561	0.554660313494281	0.554660313494281	0
1.64593061700032	1.64593061700032	0.477287293511386	0.477287293511386	0
1.66979498167589	1.66979498167589	0.393530786546305	0.393530786546305	0
1.6894715210032	1.6894715210032	0.303598812747735	0.303598812747735	0
1.70465146164059	1.70465146164059	0.20776513196292	0.20776513196292	0
1.71503971823874	1.71503971823874	0.106368080537715	0.106368080537715	0
1.72035812226562	1.72035812226562	-0.000191247201578662	-0.000191247201578662	0
1.72034855990554	1.72034855990554	-0.111451458544853	-0.111451458544853	0
1.7147759869783	1.7147759869783	-0.226893942402897	-0.226893942402897	0
1.70343128985816	1.70343128985816	-0.345946542815803	-0.345946542815803	0
1.68613396271737	1.68613396271737	-0.467987793085774	-0.467987793085774	0
1.66273457306308	1.66273457306308	-0.592351671998989	-0.592351671998989	0
1.63311698946313	1.63311698946313	-0.71833283832059	-0.71833283832059	0
1.5972003475471	1.5972003475471	-0.845192294894599	-0.845192294894599	0
1.55494073280237	1.55494073280237	-0.972163429305387	-0.972163429305387	0
1.5063325613371	1.5063325613371	-1.09845837420299	-1.09845837420299	0
1.45140964262695	1.45140964262695	-1.22327462709986	-1.22327462709986	0
1.39024591127196	1.39024591127196	-1.34580186674487	-1.34580186674487	0
1.32295581793471	1.32295581793471	-1.46522890109821	-1.46522890109821	0
1.2496943728798	1.2496943728798	-1.58075068049039	-1.58075068049039	0
1.17065683885528	1.17065683885528	-1.69157530876335	-1.69157530876335	0
1.08607807341712	1.08607807341712	-1.79693098507066	-1.79693098507066	0
0.996231524163583	0.996231524163583	-1.89607280955822	-1.89607280955822	0
0.901427883685672	0.901427883685672	-1.98828938735192	-1.98828938735192	0
0.802013414318076	0.802013414318076	-2.07290916713192	-2.07290916713192	0
0.69836795596148	0.69836795596148	-2.14930645305783	-2.14930645305783	0
0.590902633308588	0.590902633308588	-2.21690703189909	-2.21690703189909	0
0.480057281713634	0.480057281713634	-2.27519336089118	-2.27519336089118	0
0.366297613669075	0.366297613669075	-2.3237092660439	-2.3237092660439	0
0.25011215036688	0.25011215036688	-2.36206410533135	-2.36206410533135	0
0.132008945100312	0.132008945100312	-2.38993635634862	-2.38993635634862	0
0.0125121272828816	0.0125121272828816	-2.40707659357517	-2.40707659357517	0
-0.107841702395877	-0.107841702395877	-2.41330982628607	-2.41330982628607	0
-0.22850719371018	-0.22850719371018	-2.40853717433942	-2.40853717433942	0
-0.348934052427151	-0.348934052427151	-2.39273686548194	-2.39273686548194	0
-0.468570895701248	-0.468570895701248	-2.36596454438945	-2.36596454438945	0
-0.58686912292072	-0.58686912292072	-2.32835289033123	-2.32835289033123	0
-0.703286767437282	-0.703286767437282	-2.28011054704954	-2.28011054704954	0
-0.817292294789759	-0.817292294789759	-2.22152037511266	-2.22152037511266	0
-0.928368313545392	-0.928368313545392	-2.15293704356421	-2.15293704356421	0
-1.0360151657236	-1.0360151657236	-2.07478398408981	-2.07478398408981	0
-1.13975436492809	-1.13975436492809	-1.98754973708907	-1.98754973708907	0
-1.23913185178255	-1.23913185178255	-1.89178372491715	-1.89178372491715	0
-1.3337210380284	-1.3337210380284	-1.78809149308606	-1.78809149308606	0
-1.42312561268271	-1.42312561268271	-1.67712946533758	-1.67712946533758	0
-1.50698208504059	-1.50698208504059	-1.55050926316624	-1.55050926316624	0

Midpoint and upper and lower bound

Maximum error (min u): 4.64192318618954 , timestep= 98
Maximum error (max u): 1.96693113324025 , timestep= 99
Maximum error (min v): 3.97600121841456 , timestep= 76
Maximum error (max v): 4.19346419677218 , timestep= 99

Sensitivity analysis

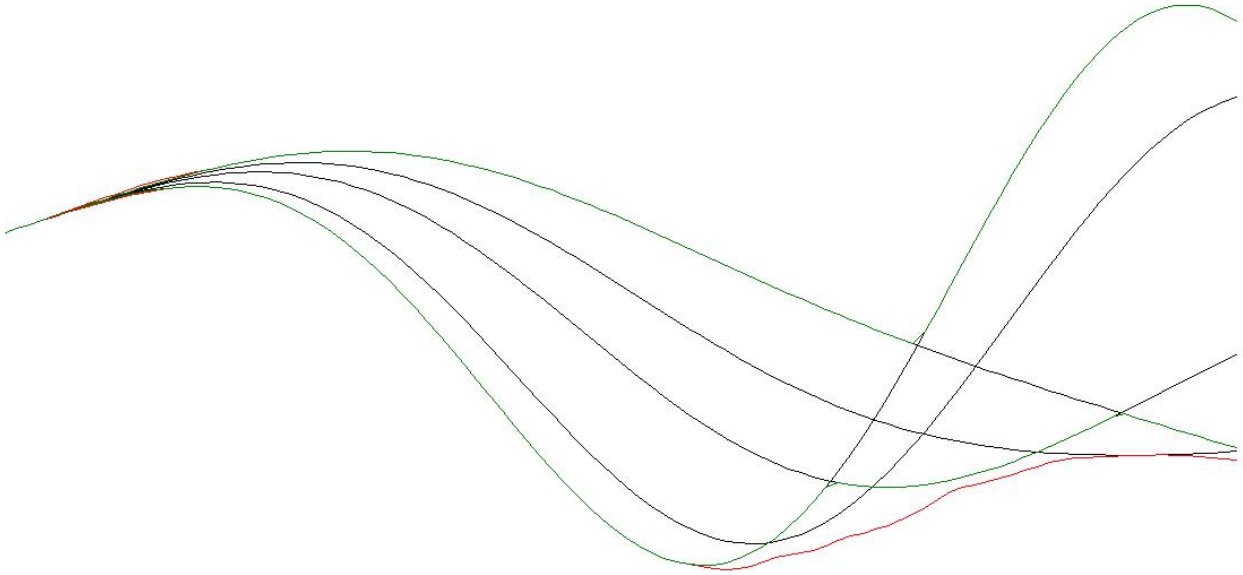
Sign vectors	Parameters
1, 1, 4 0, 0, 3 0, 1, 194 1, 0, 194	1 1 1 1.5 1.5 1 1 1



u0	u1	u2	min u (estimated)	max u (estimated)
1	1	1	1	1
1.05	1.05	1.05	1.05	1.05
1.0999875104132	1.0999937552066	1.09997502082639	1.09920654701828	1.10097502082639
1.14980018727099	1.1499000936355	1.14960037454199	1.14742495728049	1.15265037454199
1.19925123657557	1.19962561048179	1.19850253559908	1.19443959152384	1.20469851061991
1.24813043789698	1.24906507845355	1.24626199975348	1.24001506496786	1.25674135016967
1.29620546753168	1.29810199261457	1.29241686403933	1.28389775276508	1.30838827849934

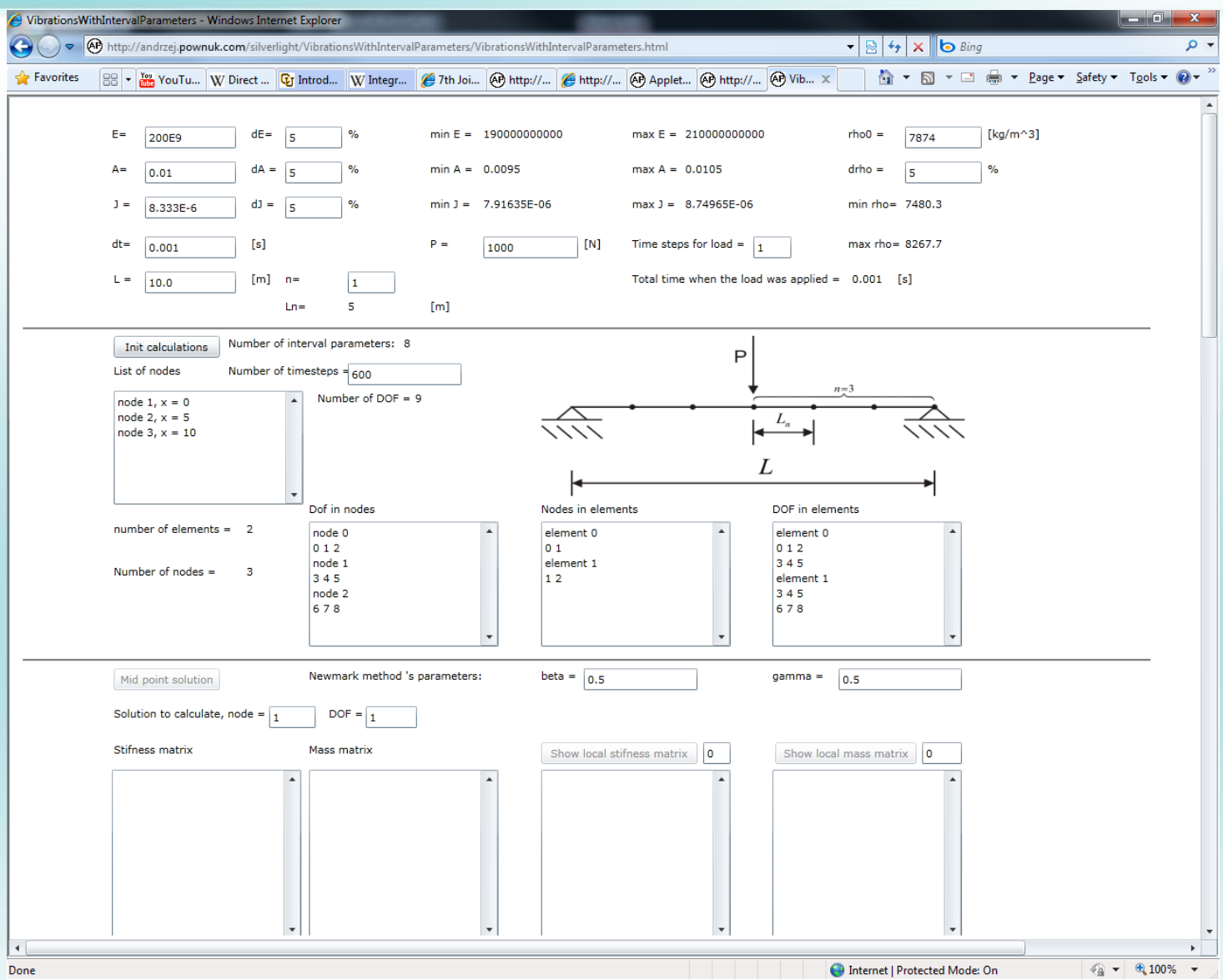
5 solutions

Maximum error (min u): 0.0403654664834101 , timestep= 16 , minKu=1.6 , minMu=1
Maximum error (max u): 0.710534736221709 , timestep= 66 , maxKu=1 , maxMu=1.4
Maximum error (min v): 1.55832146901343 , timestep= 91 , minKv=1.6 , minMv=1
Maximum error (max v): 0.673237177117273 , timestep= 48 , maxKv=1.6 , maxMv=1

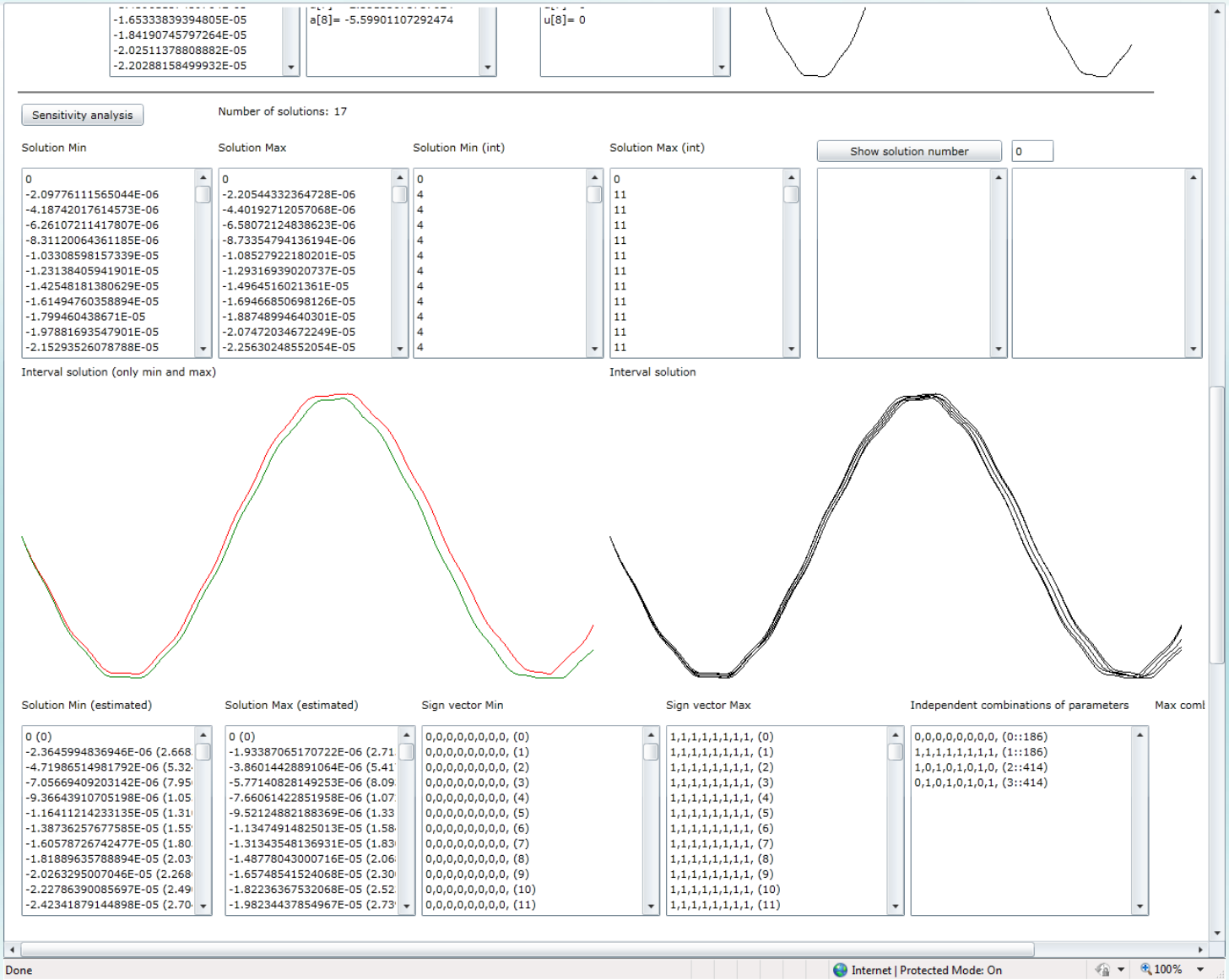


u0	u1	u2	u3	u4
1	1	1	1	1
1.05	1.05	1.05	1.05	1.05
1.0999875104132	1.0999937552066	1.09997502082639	1.09998001666111	1.09998001666111
1.14980018727099	1.1499000936355	1.14960037454199	1.14968029963359	1.14968029963359
1.19925123657557	1.19962561048179	1.19850253559908	1.19880200849592	1.19880200849592
1.24813043789698	1.24906507845355	1.24626199975348	1.24700924013574	1.24700924013574
1.29620546753168	1.29810199261457	1.29241686403933	1.29393159398913	1.29393159398913
1.34322351010891	1.34660924494616	1.33646709640096	1.33916725334161	1.33916725334161
1.38891314448551	1.39444992252108	1.37787945537882	1.38228655506728	1.38228655506728
1.43298648686022	1.44147822192651	1.41609301242141	1.42283601461697	1.42283601461697
1.47514157129439	1.48754047138912	1.45052522202847	1.46034276781947	1.46034276781947
1.51506494527608	1.53247625095666	1.48057847810275	1.49431938618698	1.49431938618698
1.55243445563309	1.57611960008675	1.50564708867402	1.52426901796891	1.52426901796891
1.58692219801311	1.61830030109624	1.52512459562716	1.549690803224	1.549690803224
1.61819760132561	1.65884522614139	1.53841136128005	1.57008550772172	1.57008550772172
1.64593061700032	1.69757973473881	1.54492233966931	1.58496131757814	1.58496131757814
1.66979498167589	1.73432910830563	1.5440949472605	1.59383973421034	1.59383973421034
1.6894715210032	1.76892000780115	1.53539694554185	1.59626150748302	1.59626150748302
1.70465146164059	1.80118194029342	1.51833424661541	1.59179254384168	1.59179254384168
1.71503971823874	1.83094872015757	1.4924585524878	1.58002972579164	1.58002972579164

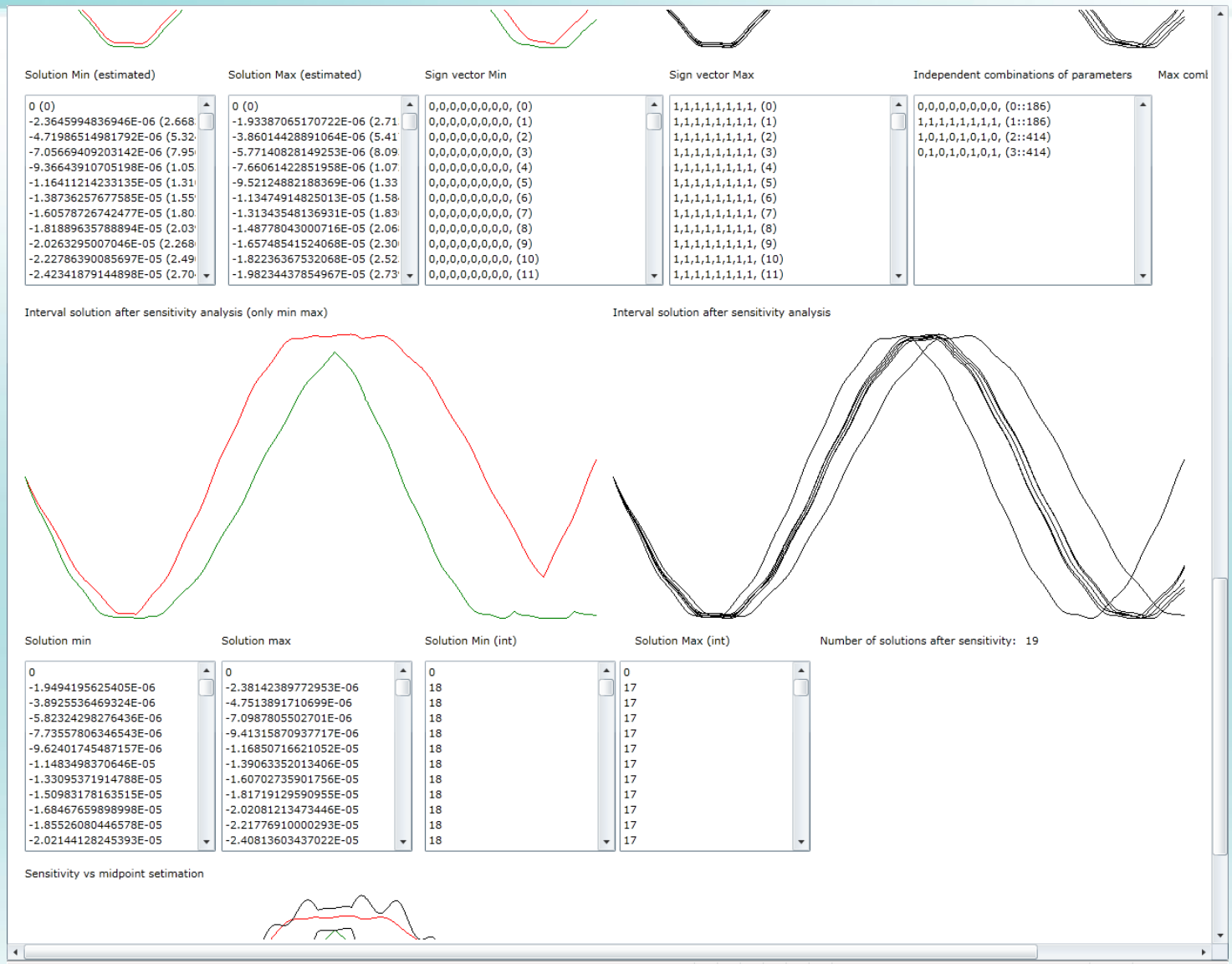
Vibrations with the interval parameters



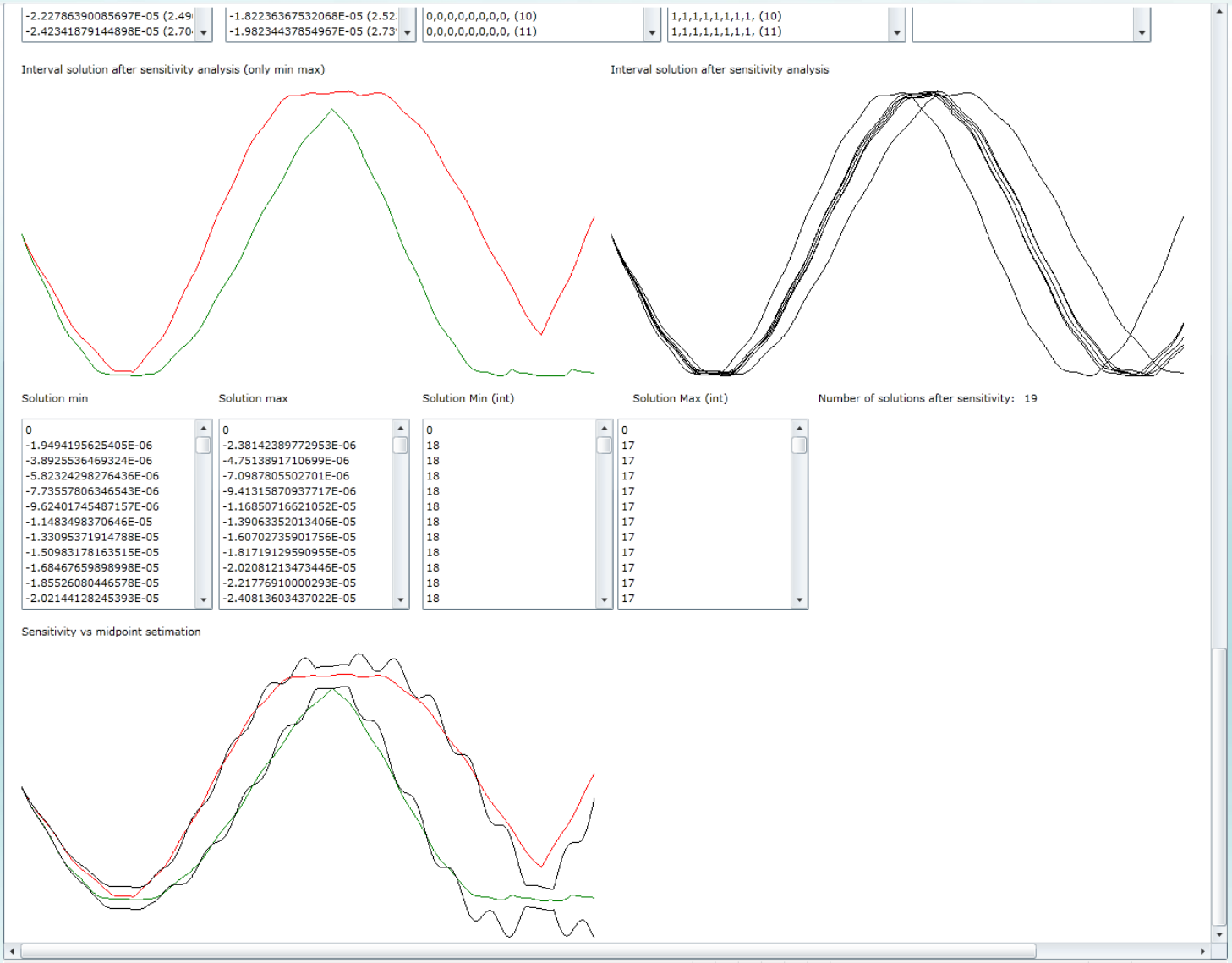
Sensitivity analysis



Interval solution after sensitivity analysis



Sensitivity vs midpoint solution



- M.V.Rama Rao, Andrzej Pownuk, Stefan Vandewalle, David Moens Transient Response of Structures with Uncertain Structural Parameters. Journal of Structural Safety (submitted for publication).

Conclusions

- Using adaptive Tylor series it is possible to get the interval solution.
- The procedure is very effective.
- It is possible to get error estimation and control the accuracy of the calculations.